Things to know for the Core Algebra Regents

Equations: Separate into 2 equations - one w/the original equation (without absolute value), the other with every term inside the absolute value negated. CHECK all solutions in the original equation. REJECT those that don't work.

ABSOLUTE VALUE

Answer: {2, 0}

3+0=3

|-3| + 0 = 3

Inequalities: Solve as if it were an equation (see above), then instead of checking the two solutions (let's call them a and b), graph them on a number line. To remember: > (greater than) shades in 2 directions, < (less than) only shades in 1

Symbol	Number Line	Write Solution as
>	€ a b	x < a OR x > b
<u>></u>	a b	x <u><</u> a OR x <u>></u> b
<	$\stackrel{\longleftarrow}{\underset{a}{\longleftarrow}}$	a < x < b
<	←	α <u><</u> x <u><</u> b

Undefined: Forget about the numerator. Set the denominator equal to zero and solve for x. Ex) $\frac{3x-1}{x+5}$ is undefined when x + 5 = 0 meaning x = -5.

Adding/Subtracting: Find a common denominator and then multiply original fractions by what each is "missing". Add/Subtract numerators. Leave denominators alone. Ex) $\frac{x-2}{3} + \frac{x+1}{4} = \frac{4x-8}{12} + \frac{3x+3}{12} =$

FRACTIONS

Multiplying: Factor first, then cancel diagonally or up/down.

Multiply straight across to get final answer.

NOTICE: when cancelling additive inverses, leave a "-1": $\frac{x-3}{3-x}$ = -1

Ex)
$$\frac{x^2-4}{3x+6} \cdot \frac{4}{2x-4} = \frac{(x+2)(x-2)}{3(x+2)} \circ \frac{4}{2(x-2)} = \frac{4}{3}$$

Dividing: Same as multiplying except FLIP the second fraction first. Simplifying: Any terms that have "+" or "-" between them must be factored first. Then cancel. Ex) $\frac{x^2-9x+20}{4x-20} = \frac{(x-4)(x-5)}{4(x-5)}$

To determine if a relation is a function...

<u>Graphs</u>: must pass the "Vertical Line Test" (no vertical line can ever intersect the graph more than once)

Points: All x-values must be DIFFERENT in a function.

f(x) notation: If f(x) is given and we want to find "f(some number)", we substitute the number in place of x on the right side of the equation.

Ex) If
$$f(x) = 2x^2 - 3x + 1$$
, then $f(5) = 2(5)^2 - 3(5) + 1 = 36$

Domain: a list of the x-values used in the relation/function.

Range: a list of the y-values used in the relation/function.

Ex)

<u>Domain</u>: 2 ≤ x ≤ 13

Range: 1 < y < 8

FUNCTIONS

Restricted Domains: many functions have a domain of "All x". Those that don't are...1) Fractions: whatever number(s) make the fraction UNDEFINED are NOT in the domain. Ex) Domain of $\frac{4x-1}{2x+3}$ is: $x \neq \frac{-3}{2}$

- 2) <u>Square Roots</u>: the radicand must be \geq zero. Ex) Domain of $\sqrt{2x+3}$ is: $x \geq \frac{-3}{2}$
- 3) Square Root in Denominator: radicand must be > zero.

Ex) Domain of
$$\frac{4x-1}{\sqrt{2x+3}}$$
 is: $x > \frac{-3}{2}$

EXPONENTIAL GROWTH

& DECAY If something is growing at a rate of r%: $C(1+r)^{\dagger}$

If something is <u>decaying</u> at a rate of r%: $C(1-r)^{\dagger}$

where ... C = current value

r = % written as decimal (move 2 places LEFT)

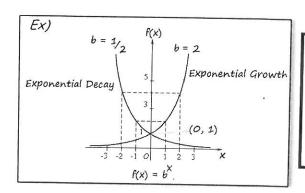
t = time

Graph of Exponential Functions: $y = b^x$

If 0 < b < 1 the graph represents DECAY (decreasing from left to right) If b > 1 the graph represents GROWTH (increasing from left to right)

(b must be positive but ≠ 1)

EXPONENT GRAPHS

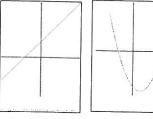


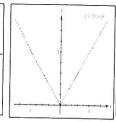
PROPERTIES of **Exponential Graphs:**

- ·Passes through (0, 1)
- ·Quadrants I and II only
- ·x axis is an <u>asymptote</u>
- (graph gets closer to but never touches x-axis)

GRAPHS

Graphs to Recognize...







LINEAR

QUADRATIC

ABS. VALUE

EXPONENTIAL

NOTE: These

rules are if x is

before symbol.

(line)

(u - shaped)

(v - shaped)

(hockey stick)

INEQUALITIES

Number lines:

REMEMBER: If divide by a negative number. the symbol switches.

- > shade RIGHT w/open circle
- < shade LEFT w/open circle
- ≥ shade RIGHT w/closed circle
- shade LEFT w/closed circle

Both x and y: These get graphed on xy-plane as lines (dotted or solid) and then a region gets shaded. (If vertical line, follow above rules)

- > dotted line/shade UP
- < dotted line/shade DOWN
- ≥ solid line/shade UP
- solid line/shade DOWN

INTERVAL NOTATION

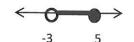
LINES

Parentheses: means UNEQUAL (use open circles)

Brackets: means EQUAL (use closed circles)

SHADE in between the 2 numbers on a # Line

Ex) Inequality notation: -3 < x < 5



in interval notation: (-3, 5]

Equation of a Line: |y = mx + b| where m = slope and b = y - interceptIf Vertical line: x = a number (slope is undefined)

If Horizontal line: y = a number (slope is zero)

Slope of a Line: (using 2 points on the line)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel lines have equal slopes.

Perpendicular Lines have negative reciprocal slopes.



A positive slope looks like...

A negative slope looks like...

To write the equation of a line: Step 1: Find slope (m). Step 2: Find y- intercept (b) by plugging a point in for (x, y) and slope in for "m" into "y = mx + b" to solve for "b".

> Ex) Write the equation of a line perpendicular to y = 2x + 7 that passes through the point (-6,4).

Step 1: since the slope of y = 2x + 7 is "2", the slope of a perp. Line would be $m = -\frac{1}{2}$.

Step 2:
$$y = mx + b$$

 $x = -6$
 $y = 4$
 $y = mx + b$
 $4 = (-\frac{1}{2})(-6) + b$
 $4 = 3 + b$
 $1 = b$
 $y = -\frac{1}{2}x + 1$

Types of Numbers:

- 1. <u>Integers</u>: a whole number that can be positive, negative, or zero.
- 2. Rational: any number that can be written as a fraction (when written as a decimal it either ENDS or REPEATS)
- 3. Irrational: a number that cannot be written as a fractions (the decimal NEVER ends and NEVER repeats)

NUMBERS

& their Properties

Properties of Numbers:

- 1. Commutative: when the numbers/variables change order Ex) 3 + 4 = 4 + 3 or $a \cdot b = b \cdot a$
 - Associative: when the parentheses change what is inside them Ex) 3 + (4 + 5) = (3 + 4) + 5 or $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- <u>Distributive</u>: the number outside () multiplies each term inside Ex) 3(4 + 5) = 3(4) + 3(5)
- 4. Identity: for Addition, can add "0" and not change the number for Multiplication, can mult by "1" and not change number Ex) 3 + 0 = 3 or $3 \cdot 1 = 3$
- 5. <u>Inverse</u>: for Addition, can add two numbers to get an answer of 0. for Multiplication, can multiply to get an answer of 1.

Ex)
$$3 + (-3) = 0$$
 or $3 \cdot \frac{1}{3} = 1$

6. Zero Property: anything multiplied by 0 is 0 Ex) $3 \cdot 0 = 0$

PARABOLAS

Equation of a Parabola: $y = ax^2 + bx + c$

If "a" is positive, graph looks like... The Vertex is a MINIMUM point

If "a" is negative, then The Vertex is a MAXIMUM point

Axis of Symmetry: (the vertical line that passes through vertex)

* This value of x should be in the middle of the table*

Vertex: First find the axis of symmetry using the formula above, then plug that x-value into the parabola's equation to find "y". Vertex = (x, y)Ex) Find the coordinates of the vertex of the parabola: $y = x^2 - 6x + 4$

Axis of Symmetry:
$$a = 1, b = -6, c = 4 \implies x = \frac{-(-6)}{2(1)} = 3$$

Vertex:
$$y = (3)^2 - 6(3) + 4 \Rightarrow y = -5$$
 VERTEX: (3, -5)

Roots: The values of x where the graph intersects the x-axis (y = 0)

A parabola can have either: 2 roots, 1 root, or no roots. See diagrams below.







To find the roots algebraically, set equation equal to zero and FACTOR. Set each factor equal to zero and solve for x.

Exponent Rules: The coefficients always perform the operation in the problem, the exponents never do.

Multiplying Problems: Coefficients Multiply Exponents ADD $Ex) 6x^6 \cdot 2x^2 = 12x^8$ **Dividing Problems:** Coefficients Divide Exponents SUBTRACT Ex) $6x^6 \div 2x^2 = 3x^4$

Adding/Subtracting Problems: Coefficients Add/Subtract Exponents STAY THE SAME Ex) $6x^6 + 2x^6 = 8x^6$ $Ex) 6x^6 + 2x^2 = 6x^6 + 2x^2$

POLYNOMIALS

Zero and Negative Exponents:

$$x^{0} = 1$$

$$x^{-n} = \frac{1}{x^{n}}$$

$$\frac{1}{x^{-n}} = x^{n}$$

Notice the Difference: $-3^2 \neq (-3)^2$ because $-3^2 = -9$ yet $(-3)^2 = +9$

Adding/Subtracting Polynomials: Only combine the "LIKE TERMS" (same variable and same exponent) Ex) $9x^3 + 7y^3 - x^3 - 6x^2 + 4y^3 = 8x^3 + 13y^3 - 6x^2$ "Subtract/From" Problems: The "from" expression goes first followed by a subtraction symbol and then the "subtract" expression in parentheses

POLYNOMIALS

QUADRATIC

EQUATIONS

Ex) Subtract
$$2x^2 + 3x - 1$$
 from $x^2 - 5x - 7$
= $(x^2 - 5x - 7) - (2x^2 + 3x - 1) = x^2 - 5x - 7 - 2x^2 - 3x + 1$
= $-x^2 - 8x - 6$

Multiplying Polynomials: Each term in the first () multiplies each term in the 2nd (). To mult binomial by binomial use FOIL (first, outer, inner, last) Ex) Find the product of 3x - 4 and x + 5.

$$= (3x - 4)(x + 5) = 3x^{2} + 15x - 4x - 20 = 3x^{2} + 11x - 20$$

Quadratic Equation: an equation that has x^2 in it. To solve:

Step 1: Get one side equal to ZERO

Step 2: If possible, FACTOR (by GCF, DOTS, or TRInomial). If not, use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
 b = c =

a = coefficient of x^2 b = coefficient of x

when

equal to 0 c = constant

Every quadratic equation has two solutions (though not necessarily real). The graph of a quadratic equation is a parabola and the solutions represent its REAL ROOTS (the places it intersects the x-axis).

Nature (Type) of Roots: to determine the type of roots a quadratic eqn. has (without actually finding them), use the Discriminant = $b^2 - 4ac$

Discriminant b² - 4ac is	Roots would be
NEGATIVE	Imaginary
ZERO	Equal (Real, Rational, & Equal)
POSITIVE PERFECT SQUARE	Rational & Unequal (Real, Rational, & Unequal)
POSITIVE but NOT a PERFECT SQUARE	Irrational (Real, Irrational, & Unequal)

Sum and Product of the Roots:

$$SUM = \frac{-b}{a} \quad PRODUCT = \frac{c}{a}$$

To write a Quadratic Equation given its roots:

ADD the Roots and MULTIPLY the Roots
$$\begin{array}{c}
X^2 + A \times Y + A \times$$

SUM:
$$(4 - 2\sqrt{5}) + (4 + 2\sqrt{5}) = 8$$
 (but use -8 in equation)
PRODUCT: $(4 - 3\sqrt{5})(4 + 3\sqrt{5}) = 16 - 9(5) = -29$ (actually use the -29)
Answer: $x^2 - 8x - 29 = 0$

<u>Completing the Square</u>: another way of solving a quadratic equation Step 1: Divide by the number in front of x^2 . Step 2: Keep all terms with x on the left and move constant to right side of =. Step 3: Set up a place holder on each side to fill in with the missing number $\left(\frac{b}{2}\right)^2$. Step 4: Rewrite the left side as $(x + \frac{b}{2})^2$. Step 5: Square root both sides - this will get rid of the "squared" on left side. Include a " \pm " on right side. Step 6: Get x alone and simplify if possible.

Ex) Solve by completing the square: $2x^2 - 6x - 7 = 0$

$$x^{2} - 3x - \frac{7}{2} = 0 (Divide by 2)$$

$$x^{2} - 3x = \frac{7}{2} (Move constant)$$

$$x^{2} - 3x + \bigcirc = \frac{7}{2} + \bigcirc (place holder)$$

$$x^{2} - 3x + \frac{9}{4} = \frac{7}{2} + \frac{9}{4} (b = -3 \longrightarrow \left(\frac{-3}{2}\right)^{2} = +\frac{9}{4})$$

$$\left(x - \frac{3}{2}\right)^{2} = \frac{7}{2} + \frac{9}{4} (left side is perfect square)$$

$$\left(x - \frac{3}{2}\right)^{2} = \frac{23}{4} (simplify)$$

$$\sqrt{\left(x - \frac{3}{2}\right)^{2}} = \pm \sqrt{\frac{23}{4}} (drop the squared on left)$$

$$x = \frac{3}{2} \pm \sqrt{\frac{23}{4}} = \frac{3}{2} \pm \frac{\sqrt{23}}{2} \text{ or } \frac{3 \pm \sqrt{23}}{2}$$

QUADRATIC EQUATIONS

(continued)

Quadratic/Linear Systems: two equations will be given - one will be quadratic (having x^2) and the other linear (no x^2). Algebraically, we want to find the point(s) of intersection of their graphs. Step 1: Solve for "y" in either of the equations, if necessary. QUADRATIC Step 2: Substitute what "y" is equal to into the other equation in LINEAR place of its "y". Step 3: This new equation should only be in terms of x. Solve for x. **SYSTEMS** Step 4: For each value of x found, find its corresponding value of y Ex) $y = |x^2 - 2x - 3|$ $x = 0 \rightarrow y = 0^2 - 2(0) - 3 = -3$ x + y = -3 $x + x^2 - 2x - 3 = -3$ $x = 1 \rightarrow y = 1^2 - 2(1) - 3 = -4$ $x^2 - x = 0$ x(x - 1) = 0Answer: (0, -3) and (1, -4) x = 0 or x = 1Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169... Simplifying Radicals: find two numbers that multiply to the number under the radical where one number must be a perfect square. (If there is a coefficient, it will multiply) Ex) Adding/Subtracting: Two radicals must have the SAME number under the radical. If so, add/subtract coefficients and leave RADICALS common radical alone. Ex) $\sqrt{27} - 5\sqrt{12} = \sqrt{9}\sqrt{3} - 5\sqrt{4}\sqrt{3}$ $= 3\sqrt{3} - 5 \cdot 2\sqrt{3} = -7\sqrt{3}$ Multiplying/Dividing: Any two radicals can mult/divide (do not have to be the same). Mult/divide the coefficients and mult/divide the radicands. Ex) Multiply and express result in simplest radical form: $9\sqrt{6} \cdot 7\sqrt{3} = 63\sqrt{18} = 63\sqrt{9}\sqrt{2} = 63 \cdot 3\sqrt{2} =$ Ex) Divide: $\frac{6\sqrt{32}}{2\sqrt{2}} = 3\sqrt{16} = 3 \cdot 4 = \boxed{12}$ Arithmetic Sequence: when the pattern is ADDING. d = difference b/w terms To find the nth ARITHMETIC term: a_1 = first term $a_n = a_1 + (n - 1)d$ n = number of term asked for Ex) Find the 100^{th} term of: 3, 7, 11, 15, 19, ... Here: n = 100, $a_1 = 3$, $d = 4 \Rightarrow a_{100} = 3 + (100-1)(4) = 399$ Geometric Sequence: when the pattern is MULTIPLYING. r = ratio b/w terms (if not obvious - divide any term by the previous one) a1 = first term To find the nth GEOMETRIC term: n = number of term asked for $a_n = a_1(r)^{n-1}$ Ex) Find the 7th term of: 6, 4, $\frac{8}{3}$, $\frac{16}{9}$,... Here: n = 7, $a_1 = 6$, $r = 2/3 \implies a_7 = 6(\frac{2}{3})^{7-1} = \frac{128}{243}$ **SEQUENCES** Recursive Sequences: a term is found by knowing the term before it. The first term " a_1 " will be given along with a formula to find " a_n " given " a_{n-1} " Ex) If $a_1 = 2$ and $a_n = 5a_{n-1} + 3$, find the first 4 terms. Need to find a1, a2, a3, and a4

 a_2 = plug in n to be 2 = $5a_{2-1} + 3 = 5a_1 + 3 = 5(2) + 3 = 13$ a_3 = plug in n to be 3 = $5a_{3-1} + 3 = 5a_2 + 3 = 5(13) + 3 = 68$ a_4 = plug in n to be 4 = $5a_{4-1} + 3 = 5a_3 + 3 = 5(68) + 3 = 343$

Mean: the average

To find the Mean: add all the numbers that divide by how many ("n").

To find missing data: Use the fact that (Mean) · (n) = SUM

To find the missing number, see what's missing to get this sum.

Ex) 78, 92, 85, 97, ? Find ? if mean is 90.

 $(90) \cdot (5) = 450 = SUM \text{ (of all 5 numbers)}$

78+92+85+97 = 352 (so far, of the 4 known numbers)

450 - 352 98

<u>Median</u>: the middle number (once the data is arranged in order). If there are two numbers in the middle, find the average of them.

Mode: the number that appears MOST often

(there can be <u>no mode</u> or even <u>more than 1</u> mode)

Range: HIGHEST - LOWEST

<u>Outlier</u>: any number that is far away from the rest. When there are outliers, the MEDIAN best represents the data.

Quantitative vs. Qualitative: QUANTITATIVE = data is numbers
QUALITATIVE = data isn't numbers

<u>Univariate vs.Bivariate</u>: UNI = one set of #'s; BI = two sets of #'s Causal Relationship: where one thing actually causes the other.

Correlation: three types POSITIVE - as one increases, so does the other

NEGATIVE – as one increases, the other decreases

NONE – scatter plot does NOT look like a line

<u>Stem and Leaf Plot</u>: the first digit(s) go in front of the line, the last digit goes after the line. <u>DON'T FORGET A KEY!</u>

Ex) If data were: 83, 88, 88, 92, 100 then plot would look like...

10 0

Key: 9 2 = 92

Mean, Median, Range, & Standard Deviation on Calculator:

Step 1: Enter all data under L_1 and frequencies (if there are any) under L_2 using STAT $\[\]$ EDIT

Step 2: Calculate these measures of central tendency and dispersion by

STAT CALC 1 - VAR STATS L_1 (only if frequencies)

Mean: \overline{X}

Median: Med

Range: maxX - minX

Interquartile Range: Q3 - Q1

Standard Deviation:

 S_x (if it is a sample)

Normal Distribution (standardized data): use the "Normal Curve" on the Reference Sheet. Remember the horizontal scale is for every half of a standard deviation so either cut the given standard deviation in half and fill in all values or skip every other value and just fill in the ones with whole numbers.

STATISTICS

STATISTICS

STATISTICS	Box and Whiskers Plot: Include an equally spaced number line on bottom	
	then show MIN, Q1, Q2 (same as median), Q3, and MAX. (see calculator	
	instructions below)	
	Ex) If MIN = 50, Q1 = 70, Q2 = 75, Q3 = 90, MAX = 100 plot looks like	
	1	
	< + + + + + + + + + + + + + + + + + + +	
	50 60 70 80 90 100	
	To find Mean and Median:	
	Step 1: Clear out old data STAT CIrList L_1	
	Step 2: Enter data	
	Step 3: Find mean and median	
	(Arrow down to find Mean = \overline{X} and Median = MED) To create Box & Whiskers:	
	Find the 5 calculations (MIN, Q1, MED, Q3, MAX) using steps 2 and 3	
	above. To create plot:	
STATISTICS	Choose: ON	
On Calculator	Type: choose the 5th graph, then graph by hitting ZOOM	
	range	
	ower quartile upper quartile	
	median	
	Donner in a la Citata	
	Regressions: used to find the equation of the line of "best fit" given data Step 1: Enter data	
	Step 1. Efficiendata	
	Step 2: Create the scatterplot (Make sure x's are in L1 and y's are in L2)	
	Choose:	
	For type choose 1st graph	
	Step 3: Graph	
	Step 4: To write the equation of the line	
	LinReg (ax + b) ENTER	
	Note that "a" is the slope (m)	
	and b is the y-intercept	

	Two Inequalities: Graph lines on xy-plane then shade according to rules listed under "INEQUALITIES". The solution set is the region the graph that was shaded by both inequalities. Label it "5". Two Equations: Three ways to solve Graphically: graph each equation and find the point(s) of intersection and the solution of the solut			
01/075140	It could be a parabola and a line (if there's x^2) or two lines (if no x^2)			
SYSTEMS of Equations (=) or Inequalities (<)	Algebraically using SUBSTITUTION: used when the system is Quadratic/Linear (one equation has x^2) or whenever one equations has either x or y alone. Substitute whatever it is equal to into the other equation, then solve. Remember to find both x and y .			
	Algebraically using ELIMINATION: (not used in Quadratic/Linear) Multiply each equation by a number that will get the coefficients of either x or y to be the same number but with opposite signs. Then			
	add the two equations and one variable will cancel. Remember to find			
	both x and y. Ex) The algebraic method that would work best on each example is			
	$2x + 3y = 5 y = x^2 + 2x - 3 7(2x + 3y = 5)$ $x = 4y + 8 3y - 2x = 5 -3(5x + 7y = 10)$			
	use Substitution use Substitution use Elimination since x is alone since Quadratic to get y to cancel			
	<u>Cube</u> : V = s ³			
VOLUME	Rectangular Prism: V = LWH			
	Cylinder: $V = \pi r^2 h$			
*	Always start with a "Let" statement that says what x represents.			
	Consecutive Integer Problems: Consecutive Consecutive Even /Odd			
	Let $x = 1^{st}$ Let $x = 1^{st}$ $x + 1 = 2^{nd}$ $x + 2 = 2^{nd}$			
	$x + 2 = 3^{rd} \qquad x + 4 = 3^{rd}$			
WORD	Deciding "who" is x: whenever two quantities are compared to one			
PROBLEMS				
INSPERMS	another, the one at the END OF THE SENTENCE is "x". Ex) The larger of two numbers is 3 less than twice the smaller. If			
	their sum is 27, find each number.			
	SUM = 27 x = smaller # $x + (2x - 3) = 27$ $x + (2x - 3) = 27$			
	2x-3 = larger # $3x-3=27$ $2(10)-3=17 = larger #$			

3x = 30x = 10

 \Rightarrow 4x + 8 = 36

4x = 28

x = 7

7 = length; 11 = width

Ex) The width of a rectangle is 4 more than the length. If the perimeter is 36 cm., find the dimensions of the rectangle. PERIMETER = 36

2L + 2W = 36

2x + 2x + 8 = 36

2(x) + 2(x + 4) = 36

x = length

x + 4 = width