Explanation of Answers—Mathematics Section

51. **(D)** Factor each number into its prime factors:

$154 = 2 \cdot 7 \cdot 11$

$196 = 2 \cdot 2 \cdot 7 \cdot 7$

The common prime factors are 2 and 7:

$154 = \boxed{2} \cdot \boxed{7} \cdot 11$

$196 = \boxed{2} \cdot 2 \cdot \boxed{7} \cdot 7$

The greatest common factor (GCF) is the product of the common prime factors:

$GCF = 2 \cdot 7 = 14$

52. **(F)** Let x represent the amount of money that Christine has in her account. Marylou has twice as much as Christine, so Marylou has $2x$ in her account. Katy has three times as much as Christine, so Katy has $3x$ in her account. Together, they have \$3,600, so $x + 2x + 3x = \$3,600$. Solve for x:

$$x + 2x + 3x = \$3,600$$
$$6x = \$3,600$$
$$x = \$600$$

53. **(D)** The ratio of XY: XZ is 1:3 and XY = 10 in. Set up a proportion to find XZ:

$$\frac{XY}{XZ} = \frac{1}{3}$$
$$\frac{10}{XZ} = \frac{1}{3}$$
$$10 \cdot 3 = XZ$$
$$30 = XZ$$

Solve for YZ:

$$YZ = XZ - XY$$
$$= 30 - 10$$
$$= 20$$

Set up another proportion to find WY:

$$\frac{WY}{YZ} = \frac{5}{2}$$
$$\frac{WY}{20} = \frac{5}{2}$$
$$WY \cdot 2 = 5 \cdot 20$$
$$WY = 50$$

Solve for WX:

$$WX = WY - XY$$
$$= 50 - 10$$
$$= 40$$

54. **(K)** A number in scientific notation is written as $a \times 10^b$, where $1 \le a < 10$ and b is a whole number. Options H and K are the only two options written in scientific notation:

Option H:

$2.5736 \times 10^1 \rightarrow 1 \le 2.5736 < 10$ and $b = 1$

Option K:

$2.5736 \times 10^3 \rightarrow 1 \le 2.5736 < 10$ and $b = 3$

However, only option K is numerically equivalent to 257.36×10:

$257.36 \times 10 = 2573.6$

$2.5736 \times 10^3 = 2573.6$

55. (A) The formula for the midpoint between two numbers a and b on a number line is $\frac{a+b}{2}$. Use the formula to find the location of point O:

$$\begin{aligned} \text{midpoint} &= \frac{a+b}{2} \\ M &= \frac{L+O}{2} \\ -3 &= \frac{-12+O}{2} \\ -6 &= -12+O \\ 6 &= O \end{aligned}$$

The formula for the distance between two points is $b-a$. Use the formula to find the length of $\overline{NO}$:

$$\begin{aligned} \text{distance} &= b-a \\ &= O-N \\ &= 6-3 \\ &= 3 \end{aligned}$$

56. (H) The sum of all six sides of the hexagon is 135, so $2(3x)+3(2x)+15=135$. Solve for x:

$$\begin{aligned} 2(3x)+3(2x)+15 &= 135 \\ 6x+6x+15 &= 135 \\ 12x &= 120 \\ x &= 10 \end{aligned}$$

57. (B) Substitute the values of x and y into the expression:

$$\begin{aligned} 2|x|-9|y| &= 2|-8|-9|-3| \\ &= 2(8)-9(3) \\ &= 16-27 \\ &= -11 \end{aligned}$$

58. (K) $(\sqrt{81})(\sqrt{25})=(9)(5)=45$

59. (C) Since there are 24 hours in a day, it will be 9:00 a.m. after 24 hours, 48 hours, and 96 hours. Therefore, 100 hours after Fernando starts thawing the turkey will be 9:00 a.m. plus 4 more hours, or 1:00 p.m.

60. (K) First, simplify the fraction:

$$\frac{0.3X}{0.25}=\frac{30X}{25}=\frac{6X}{5}$$

For $\frac{6X}{5}$ to be an integer, the product of 6 and X must end in a 0 or 5. For example:

if $6X=30$, then $\frac{6X}{5}=6$

if $6X=32$, then $\frac{6X}{5}=6.4$

if $6X=35$, then $\frac{6X}{5}=7$

The only number in the set for which $6X$ will end in a 0 or 5 is 20.0. Therefore $X=20.0$.

61. (A) All even integers have 2 as a factor. There are five even integers from 1 to 10, five even integers from 11 to 20, and so on:

1–10	2, 4, 6, 8, 10
11–20	12, 14, 16, 18, 20
...	...
121–130	122, 124, 126, 128, 130
131–135	132, 134

There are 13 consecutive groups of 10 integers. Each of these groups has five even integers, so the number of even integers from 1 to 130 is $13\times5=65$. There are two more even integers from 131 to 135. Therefore, the total number of even integers is $65+2=67$.

62. **(F)** The odd integers between 1 and 11 are 3, 5, 7, and 9. The mean of these integers is $\frac{3+5+7+9}{4}=6$.

63. **(D)** Each circle touches the center of the other circle, so the length of the rectangle is equal to the length of the diameter of one circle plus the length of the radius of the other circle: $18+9=27$ cm.

The rectangle is made up of three squares, so the side length of each square is $27 \div 3 = 9$ cm.

The width of the rectangle is equal to the side length of a square, so the area of the rectangle is $27 \times 9 = 243$ sq cm.

64. **(H)** Divide to find the number of $\frac{5}{6}$ foot pieces of wire Robyn can cut from 75 feet:

$75 \div \frac{5}{6} = 75 \times \frac{6}{5} = 90.$

65. **(E)** The total number of hours Ian worked is $2\frac{1}{4} \times 2 = 4.5$. Therefore, the total amount that he earned is $\$7.36 \times 4.5 = \$33.12$.

66. **(J)** Translate $\Psi(o,p,q,r)$ into an algebraic expression:

$\Psi(o,p,q,r) = [(o \times p) - q]^r$

Substitute the values of *o*, *p*, *q*, and *r* into the expression and solve:

$\Psi(8,2,16,3) = [(8 \times 2) - 16)]^3 = (16-16)^3 = 0^3 = 0$

$\Psi(7,7,39,2) = [(7 \times 7) - 39]^2 = (49-39)^2 = 10^2 = 100$

$\Psi(8,2,16,3) + \Psi(7,7,39,2) = 0 + 100 = 100$

67. **(B)** The number of runners who did not finish the marathon is $60 - 42 = 18$.

The fraction of runners who did not finish the marathon is $\frac{18}{60}$.

Reduce the fraction to lowest terms and then multiply by 100 to find the percentage who did not finish the marathon:

$\frac{18}{60} = \frac{3}{10} \times 100 = 30\%$

68. **(F)** Through quick estimation, we can conclude that the two consecutive positive integers are 11 and 12: $11 \times 12 = 132$. The sum of 11 and 12 is $11 + 12 = 23$.

69. **(C)** The formula for the circumference of a circle is $C = 2\pi r$. The formula for the area of a circle is $A = 2\pi r$. Substitute the formulas into the equation $s = 3t$ to solve for the radius of the circle:

$$s = 3t$$

$$2\pi r = 3(\pi r^2)$$

$$\frac{2}{3}\pi r = \pi r^2$$

$$\frac{2}{3} r = r^2$$

$$\frac{2}{3} = r$$

70. **(K)** The formula for the area of a trapezoid is $A = \frac{1}{2} \times$ height (base_1 + base_2), where base_1 and base_2 are the lengths of the parallel sides of the trapezoid. Substitute the given values into the equation and solve:

$$A = \frac{1}{2} \times \text{height } (\text{base}_1 + \text{base}_2)$$

$$= \frac{1}{2} \times 18(14+21)$$

$$= 9(35)$$

$$= 315$$

71. (B) Cross multiply and then solve for x:

$$\frac{9}{11}=\frac{x^2}{44}$$
$$11x^2=9(44)$$
$$x^2=\frac{9(44)}{11}$$
$$=9(4)$$
$$=36$$
$$x=6 \text{ or } -6$$

72. (H) Let x represent the smallest of the three numbers. In terms of x, the next two consecutive multiples of 3 are $x+3$ and $x+6$. The sum of the three numbers is 72, so $x+(x+3)+(x+6)=72$. Solve for x:

$$x+(x+3)+(x+6)=72$$
$$3x+9=72$$
$$3x=63$$
$$x=21$$

73. (C) The area of the rectangle is $16\times 4=64$ sq cm. Therefore, the area of the square is 64 sq cm, and the side length of the square is $\sqrt{64}=8$ cm. The perimeter of the square is $8\times 4=32$ cm.

74. (G) Let j represent Jeannie's height in inches:

$$\frac{1}{2}j+3=35$$
$$\frac{1}{2}j=32$$
$$j=64$$

75. (D) The residential sector consumed 30% of the total energy:

$$4{,}000\times 30\%=4{,}000\times 0.30=1{,}200$$

76. (G) The median is the middle value in a list of numbers. To find the median, list all the numbers in order from least to greatest. The number in the middle is 119.

118, 118, 118, $\boxed{119}$, 120, 121, 122

77. (C) Substitute the values of x and y into the expression:

$$\frac{12x}{x-y}=\frac{12(9)}{9-3}=\frac{108}{6}=18$$

78. (H) Corresponding angles of similar triangles have the same measure. Since triangles RSU and TSU are similar, angles R and T have the same measure.

Let x represent the measure of angle R and angle T. The sum of the angles of triangle RST is 180°, so $35+35+x+x=180$. Solve for x:

$$35+35+x+x=180$$
$$70+2x=180$$
$$2x=110$$
$$x=55$$

Let y represent the measure of $\angle SUR$. The sum of the angles of triangle RSU is 180°, so $55+35+y=180$. Solve for y:

$$55+35+y=180$$
$$90+y=180$$
$$y=90$$

79. (A)

$$5(2+q)=3(q+4)$$
$$10+5q=3q+12$$
$$2q=2$$
$$q=1$$

80. **(G)** Let x represent the original number of marbles in the jar. There are 6 blue marbles, and the probability of randomly drawing one of them is $\frac{1}{5}$. Set up a proportion to solve for x:

$$\frac{6}{x}=\frac{1}{5}$$
$$x=30$$

Let y represent the number of blue marbles Kurt added to the jar. When Kurt adds the blue marbles, the probability becomes $\frac{1}{4}$. Set up another proportion to solve for y:

$$\frac{6+y}{30+y}=\frac{1}{4}$$
$$4(6+y)=30+y$$
$$24+4y=30+y$$
$$3y=6$$
$$y=2$$

81. **(A)** Julio drives $\frac{1}{5}$ of the distance on the first day, so the fraction of the total distance that remains after the first day is $1-\frac{1}{5}=\frac{4}{5}$.

The fraction of the total distance that Julio drives on the second day is $\frac{1}{2}\times\frac{4}{5}=\frac{2}{5}$.

After two days, the fraction of the total distance that Julio has driven is $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$.

Therefore, the fraction of the total distance that Julio has left to drive is $1-\frac{3}{5}=\frac{2}{5}$.

82. **(G)** The drawing of the herb garden has a width of about 1.5 inches and a length of about 6 inches. The scale is 1 in. = 3 feet, so the width of the actual herb garden is $1.5\times3=4.5$ feet and the length is $6\times3=18$ feet.

One yard is equal to 3 feet, so the width is $4.5\div3=1.5$ yd and the length is $18\div3=6$ yd. Therefore, the area of the herb garden is $1.5\times6=9$ sq yd.

83. **(A)** If x represents the number of visitors on Friday, then the number of visitors on Saturday is expressed as $x+500$. The total number of visitors on Friday and Saturday is $x+x+500=2x+500$.

The total number of visitors on Friday and Saturday is twice the number on Sunday, so the number of visitors on Sunday is expressed as $\frac{2x+500}{2}=x+250$.

84. **(K)** The sides of triangle PSU are the radii of circle P and circle S: $\overline{SU}$ is a radius of circle S, $\overline{PU}$ is a radius of circle P, and $\overline{PS}$ is a radius of both circles. Since both circles share $\overline{PS}$, their radii are all the same length. This means that triangle PSU is an equilateral triangle, and the length of the radii of both circles is $15\div3=5$ cm.

$\overline{OQ}$ is the diameter of the circle P and a side of rectangle OQRT. Since the radius is 5 cm, OQ is $5\times2=10$ cm. The length of $\overline{OQ}$ is twice the length of $\overline{OT}$, so OT is $10\div2=5$ cm. Therefore, the area of the rectangle is $10\times5=50$ sq cm.

85. (E) Allie is now 18 years old. Her age seven years ago was $18-7=11$. Therefore, Kelly's age is $3\times 11=33$.

86. (F) The number of females was $60{,}000\times 0.53=31{,}800$. The percentage of males in the population was $100\%-53\%=47\%$, so the number of males was $60{,}000\times 0.47=28{,}200$.

Subtract to find how many more females than males lived in Utica in 2000:

$$31{,}800-28{,}200=3{,}600$$

87. (C) Divide 128 by 11 to find the number of cups Javier filled with 11 fluid ounces of lemonade. The remainder represents the amount he poured into the last cup:

$$128\div 11=11\text{R}7$$

88. (J) The area of circle M is the area of the shaded portion plus the area of circle N: $22\pi + 3\pi = 25\pi$. The formula for the area of a circle is $A = \pi r^2$. Solve for the radius:

$$\pi r^2 = 25\pi$$
$$r^2 = 25$$
$$r = 5$$

The radius of circle M is 5 cm, so its diameter is 10 cm. The diameter of circle M is equal to the side length of the square, so the area of the square is $10\times 10=100$ sq cm.

89. (C) There are 3 feet in one yard and 0.22 yard in 1 link, so the number of feet in one link is $0.22 \text{ yd}\times 3=0.66$.

Since 1 rod = 25 links and 1 link = 0.66 ft, the number of feet in 1 rod is $25\times 0.66=16.5$.

90. (F) Substitute the value of x into the expression and solve for y:

$$\frac{4(5)+6y}{2y}=8$$
$$\frac{20+6y}{2y}=8$$
$$20+6y=16y$$
$$20=10y$$
$$2=y$$

91. (E) Substitute the values of r and s into the equation and solve for t:

$$\frac{r}{30}=\frac{s}{t}$$
$$\frac{6}{30}=\frac{9}{t}$$
$$6t=270$$
$$t=45$$

92. (F) Subtract to find the missing lengths:

$$\begin{aligned}OP &= LP-LO\\ &=37-28\\ &=9\text{ cm}\end{aligned}$$

$$\begin{aligned}MO &= MP-OP\\ &=20-9\\ &=11\text{ cm}\end{aligned}$$

$$\begin{aligned}NO &= MO-MN\\ &=11-3\\ &=8\text{ cm}\end{aligned}$$

93. (B) Translate the verbal description into an algebraic equation:

If the cost of the camera at Store Y is \$9 less than $\frac{2}{3}$ the cost at Store X, then $y=\frac{2}{3}x-9$.

94. (J) $\overline{PQ}$ and $\overline{QR}$ are two sides of triangle PQR. They are also the radii of the circle, so they have the same length. The perimeter of the triangle is 39 cm and PR is 21 cm, so because the sum of the three sides of the triangle equals its perimeter, you can solve for the radius, which is equal to PQ and QR:

$$21 + r + r = 39$$
$$2r = 39 - 21$$
$$2r = 18$$
$$r = 9$$

The radii PQ and QR are each 9 cm. Therefore, the circumference of the circle is $C = 9^2\pi = 81\pi$ sq cm.

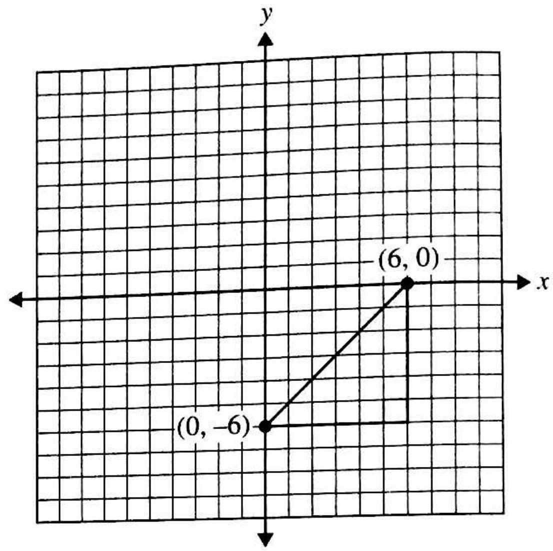
95. (A)

$$\left(\frac{3}{4} - \frac{1}{3}\right) \div \frac{5}{8} = \left(\frac{9}{12} - \frac{4}{12}\right) \div \frac{5}{8}$$
$$= \frac{5}{12} \div \frac{5}{8}$$
$$= \frac{5}{12} \times \frac{8}{5}$$
$$= \frac{40}{60}$$
$$= \frac{2}{3}$$

96. (H) Let x represent the weight of Leah's suitcase, and solve for x using the formula for the mean.

$$\frac{51 + 45 + 31 + x}{4} = 40$$
$$\frac{127 + x}{4} = 40$$
$$127 + x = 160$$
$$x = 33$$

97. (C) Look at the coordinate plane below. There are two possible coordinates for the third vertex, (0, 0) and (6, −6). Of the two, only (6, −6) is offered as an answer choice.



98. (H) There were 36 coins in the jar, and then Nicholas removed 6 quarters. This leaves 30 coins in the jar. Three of those coins are dimes, so the probability of randomly drawing a dime is $\frac{3}{30} = \frac{1}{10}$.

99. (C) Brandy painted $\frac{1}{2}$ of the wall on the first day. On the second day, she painted $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of the wall. The fraction of the wall that is painted is $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$. The fraction of the wall that is unpainted is $1 - \frac{2}{3} = \frac{1}{3}$. The ratio of painted to unpainted is $\frac{\frac{2}{3}}{\frac{1}{3}}$ or 2:1.

100. **(J)** If $3x+1$ is an odd integer, then $3x$ must be an even integer (because one less than an odd integer must be even), and x must also be an even integer (or $3x$ could not be even). For example:

$3(2)+1=7$

$3(3)+1=10$

$3(4)+1=13$

$3(5)+1=16$

You can see that $3x+1$ is only odd when an even integer is substituted for x. Substitute an even integer into each of the answer options. Only $2x+7$ produces an odd integer.