"Ratios, Rates, Proportions, and Percents"

N	AME	

Objectives:

- 1) Identify a ratio
- 2) Simplify a ratio into simplest fractional form
- 3) Identify a rate
- 4) Convert a rate into a unit rate
- 5) Solve proportions
- 6) Solve percent problems by using the percent proportion
- 7) Solve rate of increase and decrease problems
- 8) Change fractions into decimals and percents (and vice versa)
- 9) Estimate with benchmark fractions
- 10) Application problems: Similar figures, sales tax, discounts, tips, etc.

9 Ratio, Proportion, and Percent

9-1 Ratio and Rate

Objective: To write ratios as fractions in lowest terms and to write unit rates.

Terms to Know

Ratio A comparison of two numbers by division. The ratio of 2 to 3, for example, can be written in three ways:

2 to 3

2:3

Rate A ratio that compares two unlike quantities, such as miles and gallons.

Unit rate A rate that compares some quantity to 1 unit of another quantity. Miles per gallon is an example.

Write each ratio in lowest terms. Example 1

a. 4 to 10

b. 12:8

To write a ratio in lowest terms, first write it as a fraction in lowest terms. Solution

a. $\frac{4}{10} = \frac{2}{5}$ or 2 to 5 or 2:5 **b.** $\frac{12}{8} = \frac{3}{2}$ or 3 to 2 or 3:2

c. $\frac{15}{3} = \frac{5}{1}$ or 5 to 1 or 5:1

CAUTION 1 When writing ratios in lowest terms, be careful to keep two numbers in the ratio. In Example 1 (b) above, do not write $\frac{3}{2}$ as $1\frac{1}{2}$. In Example 1(c) above, do not write $\frac{5}{1}$ as 5.

Write each ratio as a fraction in lowest terms.

1. 3 to 12

3. 12:20

4. $\frac{45}{9}$

5. 32:16

6. 16 to 36

8. 8 to 4

9. 12:18

10. 15 to 6

11. 14:8

12. $\frac{16}{6}$

13. 18:24

14. $\frac{18}{27}$

15. 8 to 6

3

9-1 Ratio and Rate (continued)

CAUTION 2 Be sure that a ratio is expressed with both quantities in the same unit of measure.

Example 2 Write the ratio as a fraction in lowest terms: 5 months to 2 years

$$1 \text{ year} = 12 \text{ months}$$

so $2 \text{ years} = 24 \text{ months}$

Then write the ratio of 5 months to 24 months: $\frac{5}{24}$

The ratio of 5 months to 2 years is $\frac{5}{24}$ or 5:24 or 5 to 24.

Write each ratio as a fraction in lowest terms.

Example 3 Write the unit rate: 91 mi to 5 gal

Solution Write the comparison as a fraction.

$$\frac{\text{miles}}{\text{gallons}} \xrightarrow{\rightarrow} \frac{91}{5} = \frac{91 \div 5}{5 \div 5} = \frac{18.2}{1}$$

The rate is 18.2 mi/gal.

Write the unit rate.

Spiral Review

31. Solve:
$$\frac{3}{5}v = 12$$
 (Lesson 8-9)

34. Solve:
$$8w - 4 = -20$$
 (Lesson 4-5)

Skills and Applications of Lesson 9-2

Tell whether each proportion is True or False.

1.
$$\frac{10}{15} = \frac{2}{3}$$

2.
$$\frac{18}{24} = \frac{2}{3}$$

3.
$$\frac{24}{5} = \frac{4}{1}$$

4.
$$\frac{8}{4} = \frac{3}{1}$$

5.
$$\frac{25}{75} = \frac{1}{3}$$

6.
$$\frac{21}{6} = \frac{7}{2}$$

7.
$$\frac{15}{9} = \frac{4}{3}$$

8.
$$\frac{13}{26} = \frac{1}{2}$$

9.
$$\frac{8}{24} = \frac{1}{3}$$

10.
$$\frac{16}{12} = \frac{4}{3}$$

11.
$$\frac{2.5}{0.5} = \frac{5}{2}$$

12.
$$\frac{2.7}{1.8} = \frac{3}{2}$$

13.
$$\frac{26}{14} = \frac{2}{1}$$

14.
$$\frac{24}{21} = \frac{8}{7}$$

15.
$$\frac{12}{16} = \frac{3}{5}$$

Solve each proportion.

16.
$$\frac{4}{3} = \frac{16}{n}$$

17.
$$\frac{8}{x} = \frac{2}{5}$$

18.
$$\frac{5}{35} = \frac{a}{7}$$

19.
$$\frac{b}{6} = \frac{6}{36}$$

20.
$$\frac{5}{3} = \frac{15}{6}$$

21.
$$\frac{21}{m} = \frac{3}{2}$$

22.
$$\frac{2}{9} = \frac{z}{18}$$

23.
$$\frac{r}{24} = \frac{3}{2}$$

24.
$$\frac{1.5}{x} = \frac{4.5}{1.2}$$

25.
$$\frac{n}{25} = \frac{0.6}{3}$$

$$26. \quad \frac{4.8}{0.4} = \frac{120}{p}$$

27.
$$\frac{81}{27} = \frac{r}{0.9}$$

28.
$$\frac{6}{15} = \frac{2}{n}$$

29.
$$\frac{a}{65} = \frac{2}{5}$$

30.
$$\frac{36}{27} = \frac{z}{3}$$

31. Is
$$\frac{5}{6} = \frac{25}{30}$$
 a true proportion? Explain.

32. Is
$$\frac{4}{5} = \frac{16}{21}$$
 a true proportion? Explain.

33. A number
$$b$$
 is to 2 as 6 is to 7. Write a proportion and solve for b .

34. A number
$$n$$
 is to 9 as 5 is to 45. Write a proportion and solve for n .

3

9-4 Using Proportions

PROBLEM SOLVING

Objective: To use proportions to solve problems.

Problem

If you spent \$11.20 for 8 gal of gasoline, how much would you spend for

14 gal?

Solution

UNDERSTAND The problem is about the cost of buying gasoline.

Facts: 8 gal of gas cost \$11.20. Find: cost of 14 gal of gasoline

PLAN First set up a "word ratio" that describes what measures are being compared. Then let c be the cost of 14 gal and write a proportion. Finally,

solve the proportion using cross products.

WORK word ratio given rate desired rate

$$\frac{\text{dollars}}{\text{gallons}} \rightarrow \frac{11.20}{8} = \frac{c}{14}$$

$$11.20(14) = 8(c)$$

$$156.80 = 8c$$

$$\frac{156.80}{8} = \frac{8c}{8}$$

19.60 = c

ANSWER The cost of 14 gal of gasoline is \$19.60.

CAUTION Be sure that the numbers in your number ratio are in the same order as the words in your word ratio.

Solve using a proportion.

- 1. James bought 2 pairs of shoes for \$62.50. How many pairs of shoes can he buy for \$94?
- 2. Walter babysat for 2 h and earned \$9. How many hours must he babysit in order to earn \$22.50?
- 3. Serena Wilson paid a tax of \$288 on a house assessed at \$48,000. Using the same tax rate, find the tax on a house assessed at \$59,000.
- 4. Hisako traveled 204 mi in 6 h. How long will it take her at the same rate to travel 323 mi?
- 5. A picture $2\frac{1}{2}$ in. wide and $3\frac{1}{2}$ in. high is to be enlarged so the height will be $5\frac{1}{4}$ in. How wide will the picture be?
- **6.** Benito saves \$6 out of every \$20 he earns. He earned \$90 one month. How much did he save?

DATE	
DAIL	

Skills and Applications of Lesson 9-4

Solve using a proportion.

- 1. Duane paid \$2.24 for 8 apples. Joanne bought 5 apples at the same rate. How much did Joanne pay for the apples?
- 2. The ratio of boys to girls in a science class is 2:3. There are 18 girls in the class. How many students in the class are boys?
- 3. The cost of sending 20 pages using a commercial fax machine is \$18. At this rate, how much will it cost to send 30 pages using the same fax machine?
- 4. A recent school bond passed with 3 out of every 4 votes in favor of the bond. A total of 2550 people voted against the bond. How many people voted in favor of the bond?
- 5. During the first day of a trip, Mario drove 168 mi and used 6 gal of gasoline. At this rate, how many gallons of gasoline will Mario use for a 588-mile trip?
- 6. Isabel worked 32 h last week and was paid \$192. At the same rate, what will Isabel be paid if she works 40 h this week?
- 7. The ratio of flour to sugar in a recipe is 3 to 2. The recipe calls for $1\frac{1}{2}$ cups of flour. How many cups of sugar are needed?
- 8. Five compact cars were sold for every three sedans sold at a car dealership last month. A total of 21 sedans were sold last month. How many compact cars were sold?
- 9. The ratio of acid to water in a mixture is 3 to 7. The mixture contains 30 mL of acid. How much water does the mixture contain?

9

Practice 114

Skills and Applications of Lesson 9-5

Write each fraction, mixed number, or decimal as a percent.

1.
$$\frac{3}{5}$$

2.
$$\frac{4}{25}$$

3.
$$\frac{7}{16}$$

4.
$$\frac{20}{32}$$

5.
$$\frac{11}{12}$$

6.
$$\frac{48}{64}$$

7.
$$\frac{12}{32}$$

8.
$$\frac{3}{20}$$

9.
$$\frac{7}{25}$$

10.
$$\frac{7}{200}$$

17.
$$3\frac{3}{16}$$

18.
$$2\frac{9}{36}$$

20.
$$1\frac{8}{25}$$

Write each percent as a decimal.

28.
$$\frac{5}{8}$$
 %

Write each percent as a fraction or mixed number in lowest terms.

30.
$$\frac{7}{8}$$
%

31.
$$\frac{15}{16}$$
%

- 35. Brian made 5 out of 8 goal attempts in Saturday's soccer game. What percent of the goal attempts did Brian make?
- 36. Mei completed 17 out of 20 problems for a chemistry assignment. What percent of the chemistry assignment did Mei complete?
- 37. Out of 400 students, 96 prefer basketball to football. What percent of the students prefer basketball?

9-6 Finding a Percent of a Number

Objective: To find a percent of a number and to estimate percents of numbers.

Example 1 Find 62% of 150.

Solution To find a percent of a number, you multiply.

62% of 150
$$\downarrow$$
 \downarrow \downarrow 0.62 • 150 = 93

Find each number.

1. 48% of 85

2, 2% of 375

3. 30% of 5000

4. 6.5% of 82

5. 100% of 68

6. $9\frac{3}{4}\%$ of 672

Find each answer.

- 7. What number is 29% of 140?
- 8. What number is 10% of \$20?
- 9. 1% of 4275 is what number?

Example 2 What number is 40% of 25?

Solution Write an equation. Let *n* be the unknown number.

What number is 40% of 25? $\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$ $n \qquad = \frac{40}{100} \cdot 25$ $n = \frac{2}{5} \cdot 25 = 10$

So, 10 is 40% of 25.

Example 3 Estimate: **a.** $37\frac{1}{2}\%$ of \$160

b. 23% of 79

Solution

To estimate, use compatible numbers that are close to the given numbers.

a. $37\frac{1}{2}\%$ of $163 \leftarrow 163$ is about 160.

$$\frac{3}{8} \cdot 160 = \frac{480}{8} = 60$$
So $37\frac{1}{2}\%$ of 163 is about 60

So $37\frac{1}{2}\%$ of 163 is about 60.

b. 23% of $79 \leftarrow 23\%$ is about 25%. 79 is about 80.

$$\frac{1}{4} \cdot 80 = \frac{80}{4} = 20$$

So 23% of 78 is about 20.

(q)

9-6 Finding a Percent of a Number (continued)

Equivalent Percents, Decimals, and Fractions

$20\% = 0.2 = \frac{1}{5}$	$25\% = 0.25 = \frac{1}{4}$	$12\frac{1}{2}\% = 0.125 = \frac{1}{8}$	$16\frac{2}{3}\% = 0.1\overline{6} = \frac{1}{6}$		
$40\% = 0.4 = \frac{2}{5}$	$50\% = 0.5 = \frac{1}{2}$	$37\frac{1}{2}\% = 0.375 = \frac{3}{8}$	$33\frac{1}{3}\% = 0.\overline{3} = \frac{1}{3}$		
$60\% = 0.6 = \frac{3}{5}$	$75\% = 0.75 = \frac{3}{4}$	$62\frac{1}{2}\% = 0.625 = \frac{5}{8}$	$66\frac{2}{3}\% = 0.\overline{6} = \frac{2}{3}$		
$80\% = 0.8 = \frac{4}{5}$		$87\frac{1}{2}\% = 0.875 = \frac{7}{8}$	$83\frac{1}{3}\% = 0.8\overline{3} = \frac{5}{6}$		
100% = 1					

Estimate. The chart above may be helpful.

11.
$$62\frac{1}{2}\%$$
 of 238

15.
$$87\frac{1}{2}\%$$
 of 16

16.
$$33\frac{1}{3}\%$$
 of 61

18.
$$16\frac{2}{3}\%$$
 of 49

Spiral Review

- 19. Monte paid \$84 for 3 art lessons. At this rate, how much would he pay for 11 lessons? (Lesson 9-4)
- **20.** Solve F = ma for m. (Lesson 8-10)
- 21. What number is 18% of 40? (Lesson 9-6)

10)		
NAME	DATE	

Skills and Applications of Lesson 9-7

Find each answer.

- 1. What percent of 50 is 12?
- **3.** What percent of 900 is 198?
- 5. What percent of 1250 is 100?
- 7. What percent of 88 is 11?
- 9. What percent of 120 is 18?
- **11.** What percent of 770 is 77?
- **13.** What percent of 99 is 33?
- **15.** 16 is what percent of 64?
- **17.** 2.5 is what percent of 150?
- 19. 12.2 is what percent of 48.8?
- 21. 56 is what percent of 160?
- 23. 9 is what percent of 24?

- 2. What percent of 12 is 3?
- 4. What percent of 100 is 33?
- 6. What percent of 500 is 450?
- 8. What percent of 900 is 180?
- 10. What percent of 256 is 64?
- 12. What percent of 250 is 125?
- 14. What percent of 1000 is 890?
- 16. 10 is what percent of 25?
- 18. 6.25 is what percent of 18.75?
- 20. 37.5 is what percent of 150?
- 22. 4 is what percent of 25?
- 24. 42 is what percent of 210?
- 25. Jerry purchased a mountain bike for \$525. The sales tax he paid was \$42. What was the sales tax rate?
- 26. Gildoda Department Store expected to sell 150 suits during a sale. Actual sales were 120. What percent of the expected sales did the store actually sell?
- 27. Elena answered 47 items correctly on a 50-item test. What percent of the items did she get correct?
- 28. A high school play was attended by a total of 1250 adults and children. The audience contained 75 adults. What percent of the audience were adults?

Skills and Applications of Lesson 9-8

Find each answer.

- 1. 15% of what number is 129?
- 3. 70% of what number is 147?
- 5. 85% of what number is 153?
- 7. $66\frac{2}{3}\%$ of what number is 280?
- 9. 250% of what number is 20?
- 11. 2% of what number is 3?
- 13. 10% of what number is 9?
- 15. 9 is $\frac{1}{2}$ % of what number?
- 17. 35 is 35% of what number?
- 19. 84 is 7% of what number?
- **21.** 0.25 is $\frac{1}{8}$ % of what number?
- 23. 60 is 40% of what number?
- 25. 8 is 25% of what number?

- 2. 20% of what number is 32?
- 4. 15% of what number is 135?
- **6.** $33\frac{1}{3}\%$ of what number is 420?
- 8. 120% of what number is 240?
- 10. 150% of what number is 36?
- 12. 5% of what number is 825?
- 14. 80% of what number is 36?
- 16. 2.6 is 20% of what number?
- 18. 441 is 9% of what number?
- 20. 10 is 40% of what number?
- **22.** 21 is $37\frac{1}{2}\%$ of what number?
- 24. 12 is 75% of what number?
- 26. 4 is $\frac{1}{4}$ % of what number?
- 27. Mana got a score of 90% on her mathematics test. She got 54 items correct. How many items were on the test?
- 28. A shirt is marked down for sale. The sale price is \$24 and is 80% of the regular price. Find the regular price.
- 29. Cary earns 5% commission on all real estate sales. How much were her sales for last month if her commission was \$12,500?
- 30. A glass of skim milk contains 9 grams of protein. This is 20% of the Recommended Daily Allowance (RDA) for protein. Find the number of grams of protein in the RDA for protein.



Skills and Applications of Lesson 9-9

Find the percent of increase.

1. original number of students: 50; new number of students: 58

2. original price: \$75; new price: \$90

3. original salary: \$625; new salary: \$750

4. original depth: 8 ft; new depth: 12 ft

5. original length: 20 cm; new length: 24 cm

6. original width: 15 cm; new width: 20 cm

7. original fare: \$160; new fare: \$180

8. original cost: \$64; new cost: \$80

Find the percent of decrease.

9. original amount: \$80; new amount: \$68

10. original score: 125; new score: 75

11. original price: \$18; new price: \$16

12. original salary: \$500; new salary: \$450

13. original height: 27 in.; new height: 18 in.

14. original distance: 400 km; new distance: 320 km

15. original width: 36 mm; new width: 33 mm

16. original length: 48 in.; new length: 42 in.

- 17. The original price of a shirt was \$32. The new price is \$28. Find the percent of decrease.
- 18. In June, Gary's Appliance Store sold 60 telephones. In July the store sold 72 telephones. Find the percent of increase in the number of sales of telephones.
- Last year Shani made 12 soccer goals. This year she made 10 soccer goals. Find the percent of decrease in the number of goals Shani scored.
- 20. The Gonzalez family increased the size of their family room from 180 sq ft to 240 sq ft. Find the percent of increase in the family room area.

(13)

Practice 120

Skills and Applications of Lesson 9-10

Find each answer using a proportion.

- 1. What number is 5% of 290?
- 3. What number is 15% of 104?
- 5. What number is 65% of 2080?
- 7. 5% of 740 is what number?
- 9. 20% of 5680 is what number?
- 11. 28 is 50% of what number?
- 13. 2416 is 40% of what number?
- 15. 53 is 2% of what number?
- 17. $66\frac{2}{3}$ % of what number is 22?
- 19. 15 is what percent of 60?
- 21. 375 is what percent of 500?
- 23. 35 is what percent of 105?
- 25. What percent of 320 is 32?

- 2. What number is 4% of 10?
- 4. What number is 75% of 1204?
- 6. What number is 35% of 2004?
- 8. 13% of 1800 is what number?
- 10. 16% of 2000 is what number?
- **12.** 2 is 1% of what number?
- 14. 52 is 5% of what number?
- 16. 63 is 9% of what number?
- 18. $\frac{1}{2}$ % of what number is 3?
- 20. 40 is what percent of 1000?
- 22. 425 is what percent of 425?
- 24. 10 is what percent of 15?
- **26.** What percent of 92 is 23?
- 27. Last week the Florente family spent \$102 on food. This represents 12% of Mr. Florente's income for the week. How much does Mr. Florente earn per week?
- 28. Jake purchased 2 new tires for his car. The total price including sales tax was \$96.30. The price of the tires was \$90. What percent sales tax did Jake pay?
- 29. Aisha had an inventory of 580 books. She donated 5% of her books to a charity. How many books did she donate?
- 30. Julio sells skateboards at a local shop and earns 12% commission on all skateboards sold. Last week he sold skateboards worth a total of \$920. How much did Julio earn in commissions?



Practice 121 (for use after Lesson 9-10)

Nonroutine Practice: Shopping Mix-Up

There has been a mix-up in shopping bags and receipts. Each person knows only the total amount spent and each bag contains only the items purchased with the discount marked on the tags. Everyone paid 5% sales tax, calculated after the discount. Match the shopper with his or her purchase.

Shop	pers:	•			
	1.	Sarah spent \$30.45	2.	Kenji spent \$17.00	
	3.	Jack spent \$29.40	4.	Rebekah spent \$19.32	
	5.	Solveig spent \$4.41	6.	Alan spent \$28.98	
	<u> </u>	Rueben spent \$13.23	8.	Marie spent \$27.72	
	9.	Dao spent \$17.85	10.	Lynette spent \$28.56	
Item	s purch	ased:			
A.	three pairs of socks for \$1.75 each, with a 20% discount				
B.	a camera listed for \$18 marked 20% off, and a roll of film for \$1.79				
C.	sunglasses for \$7 and a hat for \$11, both marked 30% off				
D.	a shirt marked \$28 and a sweater marked \$30, both on sale at 50% off				
E.	two shirts each marked \$22, on sale at a 40% discount				
F.	a pair of shoes discounted 30% which originally sold for \$40				
G.	a pair of jeans marked \$32 on sale at 15% off				
Ħ.	a wreath for \$16, discounted 10%, and a spool of ribbon for \$4				
I.	a soccer ball listed at \$20, sold at 15% off				

J. a jacket listed at \$46, sold at 40% off

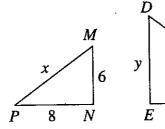
15)

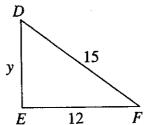
Practice 131

Skills and Applications of Lesson 10-8

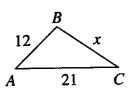
Find the values of x and y.

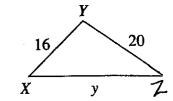
1. $\triangle MNP \sim \triangle DEF$



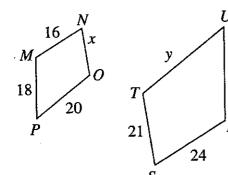


2. $\triangle ABC \sim \triangle XYZ$

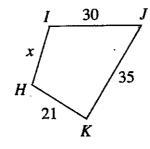


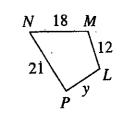


3. quad. MNOP ~ quad. RSTU



4. quad. HIJK ~ quad. LMNP





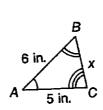
- 5. $\triangle ABC \sim \triangle XYZ$. The lengths of the sides of $\triangle ABC$ are 8 in., 12 in., and 15 in. The length of the longest side of $\triangle XYZ$ is 30 in. What are the lengths of the other two sides of $\triangle XYZ$?
- 6. The width of a rectangle is 6 cm, and its area is 96 cm². The width of a similar rectangle is 15 cm. What is the height of the similar rectangle?
- 7. The sides of a regular pentagon are each 8 cm, and its area is 110 cm². Can you determine the length of a side of a similar pentagon? Explain.
- 8. If $\triangle ABC \sim \triangle XYZ$, then is $\triangle ABC \cong \triangle XYZ$? Explain.

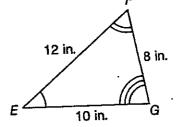
Practice Worksheet 12-9

Similar Triangles

Use the similar triangles at the right to answer Exercises 1-3.

1. List three proportions for $\triangle ABC$ and $\triangle EFG$.



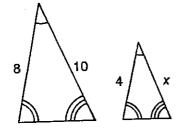


2. What side corresponds to \overline{AB} ?

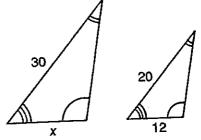
3. Find the value of x.

Find the value of x in each pair of similar triangles.

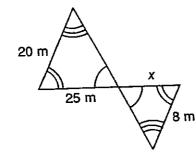
4.



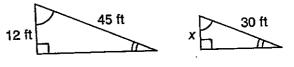
5.



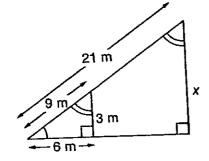
6.



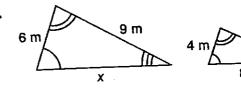
7.



8.



9.

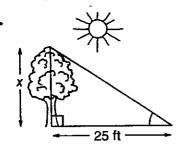


Practice Worksheet 12-10

Application: Indirect Measurement

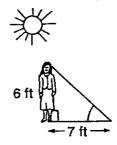
Write a proportion to find each missing measure x. Then find x.

1.



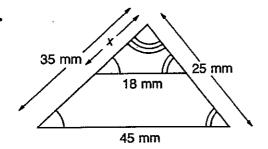
6 ft +10 ft =

2.

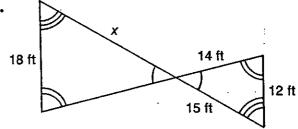


35 ft

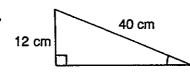
3.



4.

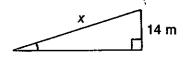


5.

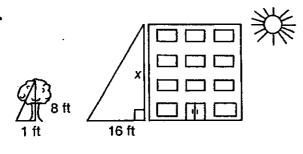


30 cm

25 m 7 m

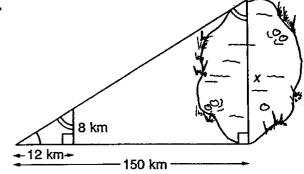


7.



8.

6.

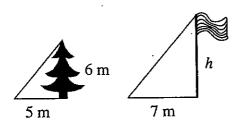




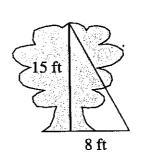
Skills and Applications of Lesson 10-9

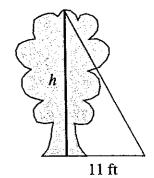
Each pair of triangles is similar. Find the unknown distance h.

1.

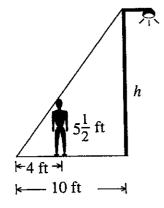


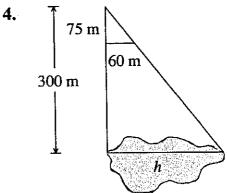
2.





3.





- 5. An oak tree casts a shadow of 14 ft. A 30 ft flagpole casts a shadow of 6 ft. How tall is the oak tree?
- 6. A 1000 ft building near the Sears Tower in Chicago casts a shadow of 400 ft. The Sears Tower is 1454 ft tall. How long a shadow does the Sears Tower cast?