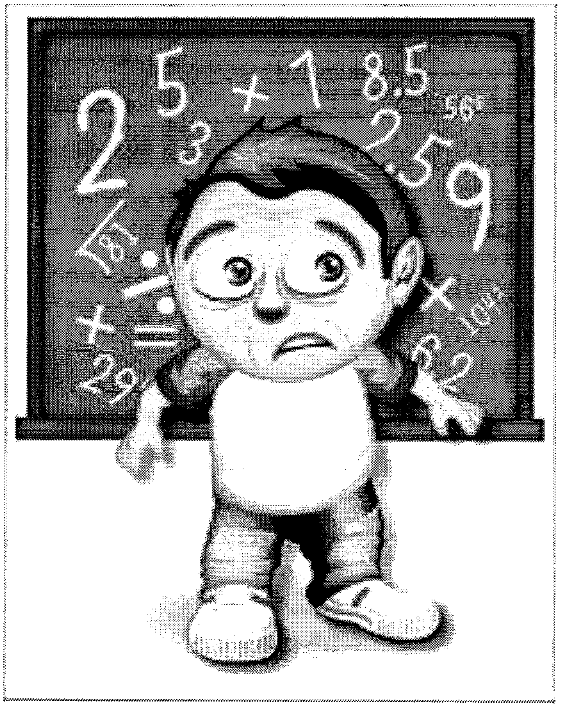
Algebra 1



Regents Review

2014

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Name Key

What to expect on JUNE 3rd…

2014 Regents Examination in Algebra I (Common Core) Design

Test Component	Number of Questions	Credits per Question	Total Credits per Section
Part I	24	2	48
Part II	8	2	16
Part III	4	4	16
Part IV	1	6	6
Total	37	-	86

Studying and Test Taking Tips

1. Always read math problems completely before beginning any calculations. If you "glance" too quickly at a problem, you may misunderstand what really needs to be done to complete the problem.

2. Whenever possible, draw a diagram. Even though you may be able to visualize the situation mentally, a hand drawn diagram will allow you to label the picture, to add auxiliary lines, and to view the situation from different perspectives.

3. If you know that your answer to a question is incorrect, and you cannot find your mistake, start over on a clean piece of paper. Oftentimes when you try to correct a problem, you continually overlook the mistake.

4. Do not feel that you must use every number in a problem when doing your calculations. Some mathematics problems have "extra" information. These questions are testing your ability to recognize the needed information, as well as your mathematical skills.

5. Be sure that you are working in the same units of measure when performing calculations. If a problem involves inches, feet AND yards, be sure to make the appropriate conversions so that all of your values are in the same unit of measure (for example, change all values to feet).

6. Be sure that your answer "makes sense" (or is logical). For example, if a question asks you to find the number of feet in a drawing and your answer comes out to be a negative number, know that this answer is incorrect. (Distance is a positive concept - we cannot measure negative feet.)

7. If time permits, go back and resolve the more difficult problems.

8. Remain confident! Focus on what you DO know, not on what you do not know.

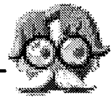
9. When asked to "show work" or "justify your answer", don't be lazy. Write down EVERYTHING about the problem, including the work you did on your calculator. Include diagrams, calculations, equations, and explanations written in complete sentences. Now is the time to "show off" what you really can do with this problem.

10. If you are "stuck" on a particular problem, go on with the rest of the test. Oftentimes, while solving a new problem, you will get an idea as to how to attack that difficult problem.

11. If you simply cannot determine the answer to a question, make a guess. Think about the problem and the information you know to be true. Make a guess that will be logical based upon the conditions of the problem.

12. Believe, Achieve, Succeed!!!

Algebra – Things to Remember!



Scientific Notation:
3.2×10^{12}
The first number must be $1 \leq n < 10$

Factorial:
$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$1! = 1$
FYI: $0! = 1$

Absolute Value:
$|-5| = 5$
$|5| = 5$
Represents distance

Undefined:
$\frac{6}{7-x}$ is undefined when $x = 7$ since the denominator = 0.

Multiply: (distribute or FOIL)
$(x+3)(x+2) = x \cdot x + x \cdot 2 + 3 \cdot x + 3 \cdot 2$
$= x^2 + 5x + 6$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Add Fractions:
Get the common denominator:
$\frac{5x}{6} + \frac{3x}{2} = \frac{5x}{6} + \frac{9x}{6} = \frac{14x}{6} = \frac{7x}{3}$

Inequalities:
$5 - 3x \leq 13 + x$ Remember to
$-3x \leq 8 + x$ change direction
$-4x \leq 8$ of inequality when
$x \geq -2$ mult/div by a negative.

x = abscissa, y = ordinate

Slope:
$m = \frac{\textit{vertical change}}{\textit{horizontal change}} = \frac{\textit{rise}}{\textit{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Exponents:
$(-3)^2 \neq -3^2$
$2^0 = 1$
$4^{-3} = \frac{1}{4^3}$
$x^m \cdot x^n = x^{m+n}$
$(x^m)^n = x^{mn}$
$\frac{x^m}{x^n} = x^{m-n}$
$(xy)^n = x^n \cdot y^n$

Polygons and sides:
triangle – 3
quadrilateral – 4
pentagon – 5
hexagon – 6
septagon – 7
octagon – 8
nonagon – 9
decagon – 10
dodecagon - 12

Direct Variation:
$y = kx$ where k = constant of variation
$k = y/x$

Factor:
Look for a GCF (greatest common factor)
Factor binomial or trinomial.
$a^2 - b^2 = (a+b)(a-b)$

Systems:

$y - 2x = 1$ $y + 2x = 9$	*Linear:* substitute; add to eliminate one variable or graph.
$y = x^2 - x - 6$ $y = 2x - 2$	*Linear Quadratic:* substitute or graph

For inequality systems, graph.

Equations of Lines: m = slope
$y = mx + b$ slope-intercept
$y - y_1 = m(x - x_1)$ point-slope

Properties of Real Numbers:
Commutative Property: a + b = b + a ab = ba
Associative Property: a+(b+c) = (a+b)+c a(bc) = (ab)c
Distributive Property: a(b+c) = ab + ac
Identity: a + 0 = a a • 1 = a
Inverse: a + (-a) = 0 a • (1/a) = 1
Zero Property: a • 0 = 0

Degree:
Degree of monomial = sum of exponents
$4x^3$ is of degree 3
x^2y^3 is of degree 5

Solving Equations:
1. Deal with any parentheses in the problem.
2. Combine similar terms on same side of = sign.
3. Get the needed variables on the same side of = sign.
4. Isolate the needed variable by add or subtract.
5. Find the needed variable by divide or multiply.

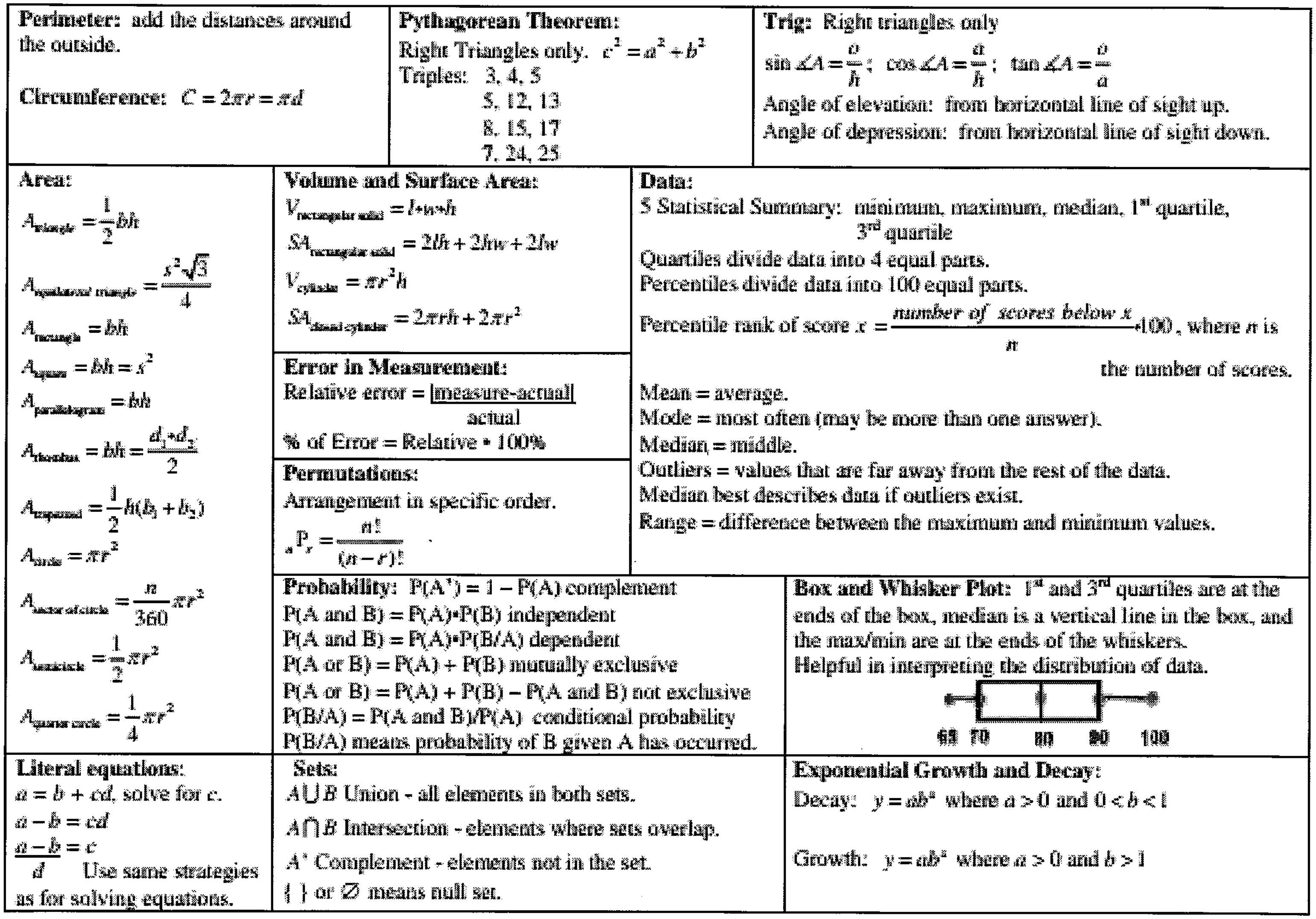
Quadratic Equation:
$x^2 - 5x + 6 = 0$ Set = 0.
$(x-3)(x-2) = 0$ Factor.
$x = 3;\ x = 2$ Find roots

Interval Notation:
$(1,5) \leftrightarrow 1 < x < 5$
$[1,5] \leftrightarrow 1 \leq x \leq 5$

Function: Passes the vertical line test. A set of ordered pairs in which each x element has only one y element associated with it.
$f(x) = 3x + 4$
$f(3) = 3 \cdot 3 + 4 = 13$

Parallel and Perpendicular:
Parallel: slopes are equal.
Perpendicular: slopes are negative reciprocals (flip over and negate)

Parabola:
$y = ax^2 + bx + c$
Axis of symmetry:
$x = \frac{-b}{2a}$
Roots: where the graph crosses the x-axis.



Perimeter: add the distances around the outside.

Circumference: $C = 2\pi r = \pi d$

Pythagorean Theorem:
Right Triangles only. $c^2 = a^2 + b^2$
Triples: 3, 4, 5
5, 12, 13
8, 15, 17
7, 24, 25

Trig: Right triangles only
$\sin \measuredangle A = \frac{o}{h}$; $\cos \measuredangle A = \frac{a}{h}$; $\tan \measuredangle A = \frac{o}{a}$
Angle of elevation: from horizontal line of sight up.
Angle of depression: from horizontal line of sight down.

Area:

$A_{triangle} = \frac{1}{2}bh$

$A_{equilateral\ triangle} = \frac{s^2\sqrt{3}}{4}$

$A_{rectangle} = bh$

$A_{square} = bh = s^2$

$A_{parallelogram} = bh$

$A_{rhombus} = bh = \frac{d_1 \cdot d_2}{2}$

$A_{trapezoid} = \frac{1}{2}h(b_1 + b_2)$

$A_{circle} = \pi r^2$

$A_{sector\ of\ circle} = \frac{n}{360}\pi r^2$

$A_{semicircle} = \frac{1}{2}\pi r^2$

$A_{quarter\ circle} = \frac{1}{4}\pi r^2$

Volume and Surface Area:

$V_{rectangular\ solid} = l \cdot w \cdot h$

$SA_{rectangular\ solid} = 2lh + 2hw + 2lw$

$V_{cylinder} = \pi r^2 h$

$SA_{closed\ cylinder} = 2\pi rh + 2\pi r^2$

Error in Measurement:
Relative error = $\frac{|measure - actual|}{actual}$
% of Error = Relative • 100%

Permutations:
Arrangement in specific order.
$_nP_r = \frac{n!}{(n-r)!}$

Data:
5 Statistical Summary: minimum, maximum, median, 1st quartile, 3rd quartile
Quartiles divide data into 4 equal parts.
Percentiles divide data into 100 equal parts.
Percentile rank of score $x = \frac{number\ of\ scores\ below\ x}{n} \cdot 100$, where n is the number of scores.

Mean = average.
Mode = most often (may be more than one answer).
Median = middle.
Outliers = values that are far away from the rest of the data.
Median best describes data if outliers exist.
Range = difference between the maximum and minimum values.

Probability: P(A') = 1 – P(A) complement
P(A and B) = P(A)•P(B) independent
P(A and B) = P(A)•P(B/A) dependent
P(A or B) = P(A) + P(B) mutually exclusive
P(A or B) = P(A) + P(B) – P(A and B) not exclusive
P(B/A) = P(A and B)/P(A) conditional probability
P(B/A) means probability of B given A has occurred.

Box and Whisker Plot: 1st and 3rd quartiles are at the ends of the box, median is a vertical line in the box, and the max/min are at the ends of the whiskers.
Helpful in interpreting the distribution of data.

Literal equations:
$a = b + cd$, solve for c.
$a - b = cd$
$\frac{a-b}{d} = c$ Use same strategies as for solving equations.

Sets:
$A \bigcup B$ Union - all elements in both sets.
$A \bigcap B$ Intersection - elements where sets overlap.
A' Complement - elements not in the set.
{ } or $\varnothing$ means null set.

Exponential Growth and Decay:
Decay: $y = ab^x$ where $a > 0$ and $0 < b < 1$

Growth: $y = ab^x$ where $a > 0$ and $b > 1$

THE UNIVERSITY OF THE STATE OF NEW YORK

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

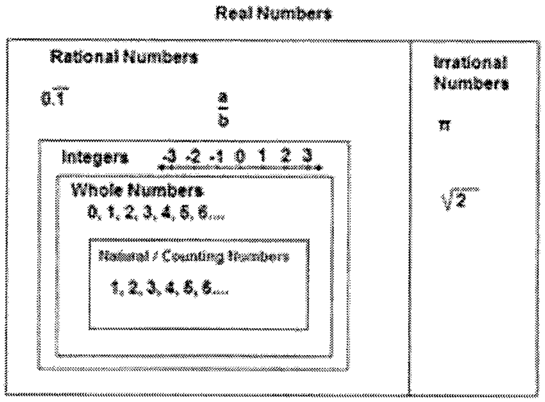
1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

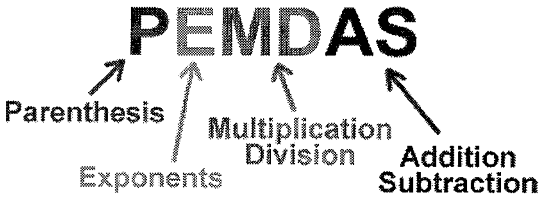
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1-r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		

REVIEW OF REAL NUMBERS, ORDER OF OPERATIONS, ABSOLUTE VALUE AND ALGEBRAIC EXPRESSIONS



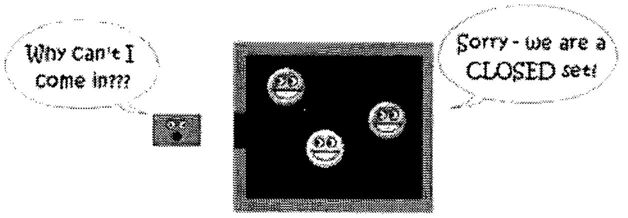


To evaluate an algebraic expression:

- Substitute the given value(s) of the variable(s).
- Use order of operations (PEMDAS) to find the value of the resulting numerical expression.

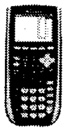
Undefined: $\frac{6}{7-x}$ is undefined when $x = 7$ since the denominator would equal zero.

A set is ***closed*** (under an operation) if and only if the operation on two elements of the set produces another element of the set. If an element outside the set is produced, then the operation is ***not closed***.



Absolute Value: is the distance that a number is away from zero on a number line.

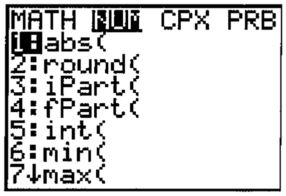
Ex: $|-9-7| = |-16| = 16$



You can find the absolute value function by pressing the **Math** key.
Arrow to the right to find the **NUM** menu.
On this screen you will find:

#1 abs(

the absolute value function.



Regents Review

1.

What is the value of the expression $2x^3y$ when $x = -2$ and $y = 3$?

(1) −192
(2) −108
[✓ (3) −48]
(4) 48

$2(-2)^3(3)$

2.

The function $y = \frac{x}{x^2 - 9}$ is undefined when the value of x is

(1) 0 or 3
[✓ (2) 3 or −3]
(3) 3, only
(4) −3, only

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3 \mid x = 3$

3.

Which value of x is the solution of $15\left(\frac{2x}{5} + \frac{1}{3} = \frac{7x - 2}{15}\right)$

(1) $\frac{3}{5}$
(2) $\frac{31}{26}$
(3) 3
[✓ (4) 7]

$6x + 5 = 7x - 2$

$-6x \qquad -6x$

$5 = 1x - 2$

$+2 \qquad +2$

$x = 7$

4.

For which value of m is the expression $\frac{15m^2n}{3 - m}$ undefined?

(1) 1
(2) 0
[✓ (3) 3]
(4) −3

5.

For which value of x is the expression $\frac{3x - 3}{x - 5}$ undefined?

(1) 1
(2) −1
[✓ (3) 5]
(4) −5

6.

If the temperature in Buffalo is 23° Fahrenheit, what is the temperature in degrees Celsius? [Use the formula $C = \frac{5}{9}(F - 32)$.]

[✓ (1) −5]
(2) 5
(3) −45
(4) 45

$C = \frac{5}{9}(23 - 32)$

$C = \frac{5}{9}(-9)$

$C = -5$

7.

What is the solution of the equation $3y - 5y + 10 = 36$?

[✓ (1) −13]
(2) 2
(3) 4.5
(4) 13

$-2y + 10 = 36$

$-10 \quad -10$

$\frac{-2y}{-2} = \frac{26}{-2}$

$y = -13$

Be careful

8. Mario paid $44.25 in taxi fare from the hotel to the airport. The cab charged $2.25 for the first mile plus $3.50 for each additional mile. How many miles was it from the hotel to the airport?

(1) 10
(3) 12
(2) 11
[✓ (4) 13]

1st mi, add. mi

$\underset{1^{st}\,mi}{2.25} + \underset{add.\,mi}{3.50X} = 44.25$

$3.50X = 42$

$X = 12$

$+1$

13

---

9. In the equation $A = p + prt$, t is equivalent to

(1) $\frac{A - pr}{p}$
(3) $\frac{A}{pr} - p$
[✓ (2) $\frac{A - p}{pr}$]
(4) $\frac{A}{p} - pr$

$A = p + prt$

$-p \quad -p$

$\frac{A-p}{pr} = \frac{\cancel{pr}t}{\cancel{pr}}$

$t = \frac{A-p}{pr}$

---

10. When solved for y, the equation $ay - b = c$ is equal to

(1) $\frac{c - b}{a}$
(3) $\frac{c + b}{y}$
(2) $\frac{c + a}{b}$
[✓ (4) $\frac{c + b}{a}$]

$ay - b = c$

$+b \quad +b$

$\frac{ay}{a} = \frac{c+b}{a}$

$y = \frac{c+b}{a}$

---

11. If $x = 2a - b^2$, then a equals

(1) $\frac{x - b^2}{2}$
(3) $\frac{b^2 - x}{2}$
[✓ (2) $\frac{x + b^2}{2}$]
(4) $x + b^2$

$X = 2a - b^2$

$+b^2 \quad +b^2$

$\frac{X+b^2}{2} = \frac{\cancel{2}a}{\cancel{2}}$

$a = \frac{X+b^2}{2}$

---

12. The equation $P = 2L + 2W$ is equivalent to

[✓ (1) $L = \frac{P - 2W}{2}$]
(3) $2L = \frac{P}{2W}$
(2) $L = \frac{P + 2W}{2}$
(4) $L = P - W$

$P = 2L + 2W$

$-2W \quad -2W$

$\frac{P-2W}{2} = \frac{2L}{2}$

$L = \frac{P-2W}{2}$

---

13. Robin spent $17 at an amusement park for admission and rides. If she paid $5 for admission, and rides cost $3 each, what is the total number of rides that she went on?

(1) 12
(3) 9
(2) 2
[✓ (4) 4]

x = # of rides

$5 + 3X = 17$

$3X = 12$

$X = 4$

REVIEW OF SOLVING LINEAR INEQUALITIES

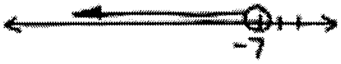
$<$ or $>$ use open circle

$\leq$ or $\geq$ use closed circle

Examples

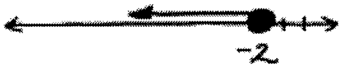
$x > 3$

$x < -7$



$x \geq 10$

$x \leq -2$



***Remember**: if you multiply/ divide by a negative number, the symbol switches.*

KEYWORDS:

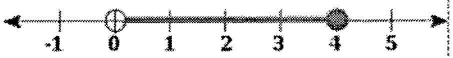
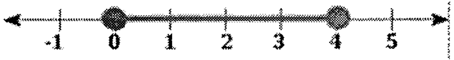
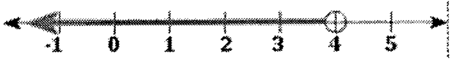
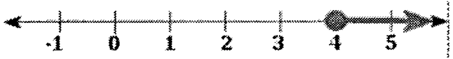
at most/maximum ≤

at least/minimum ≥

Interval Notation:

Parentheses: means UNEQUAL (OPEN CIRCLES)

Brackets: means EQUAL (CLOSED CIRCLES)

Type of Interval	Interval Notation	Graphical Notation
Open Interval	(0, 4)	
Half Open Interval	(0, 4]	
	[0, 4)	
Closed Interval	[0, 4]	
Non Ending Interval	(-∞, 4)	
	[4, +∞)	

Ex:

$$\begin{array}{r}
2x - 5 \geq 7 \\
+5 \quad +5 \\
\hline
2x \geq 12 \\
\dfrac{2x}{2} \geq \dfrac{12}{2} \\
x \geq 6
\end{array}$$

ditch the 5

ditch the 2

Ex:

$$\begin{array}{c}
-3 \leq 2x - 1 \leq 5 \\
+1 \qquad +1 \quad +1 \\
\hline
-2 \leq 2x \leq 6 \\
\dfrac{-2}{2} \leq \dfrac{2x}{2} \leq \dfrac{6}{2} \\
-1 \leq x \leq 3
\end{array}$$

ditch the -1

ditch the 2

Regents Review

1.

Which inequality is represented by the graph below?

(1) $-2 \le x \le 3$
(2) $-2 < x < 3$
(3) $-2 \le x < 3$
[✓ (4)] $-2 < x \le 3$

2.

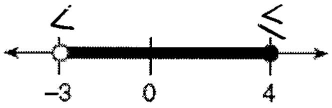
What is the value of x in the inequality $14 \ge 3x + 2$?

(1) $-4 \ge x$
(2) $-4 \le x$
[✓ (3)] $4 \ge x$
(4) $4 \le x$

$14 \geq 3x + 2$
$-2 \quad -2$
$12 \geq 3x$
$4 \geq x \Rightarrow x \leq 4$

3.

Which inequality is represented in the accompanying graph?



(1) $-3 \le x < 4$
(2) $-3 \le x \le 4$
(3) $-3 < x < 4$
[✓ (4)] $-3 < x \le 4$

4.

Which inequality is equivalent to $2x - 1 > 5$?

(1) $x > 6$
(2) $x > 2$
(3) $x < 5$
[✓ (4)] $x > 3$

$\frac{2x}{2} > \frac{6}{2}$
$x > 3$

5.

Students in a ninth grade class measured their heights, h, in centimeters. The height of the shortest student was 155 cm, and the height of the tallest student was 190 cm. Which inequality represents the range of heights?

(1) $155 < h < 190$
[✓ (2)] $155 \le h \le 190$
(3) $h \ge 155$ or $h \le 190$
(4) $h > 155$ or $h < 190$

$155 \leq h \leq 190$

6.

Mrs. Smith wrote "Eight less than three times a number is greater than fifteen" on the board. If x represents the number, which inequality is a correct translation of this statement?

[✓ (1)] $3x - 8 > 15$
(2) $3x - 8 < 15$
(3) $8 - 3x > 15$
(4) $8 - 3x < 15$

$3x - 8 > 15$
$3x \geq 23$
$x > \frac{23}{3}$

7.

Which verbal expression represents $2(n - 6)$?

(1) two times n minus six

(2) two times six minus n

(3) two times the quantity n less than six

[✓ (4) two times the quantity six less than n]

8.

An electronics store sells DVD players and cordless telephones. The store makes a \$75 profit on the sale of each DVD player (d) and a \$30 profit on the sale of each cordless telephone (c). The store wants to make a profit of at least \$255.00 from its sales of DVD players and cordless phones. Which inequality describes this situation?

(1) $75d + 30c < 255$

(2) $75d + 30c \leq 255$

(3) $75d + 30c > 255$

[✓ (4) $75d + 30c \geq 255$]

$75d + 30c \geq 255$

9.

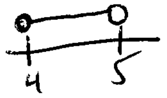
The statement "$x \geq 4$ and $2x - 4 < 6$" is true when x is equal to

(1) 1

(2) 10

~~(3) 5~~

[✓ (4) 4]



$2x-4<6$

$+4 \quad +4$

$2x<10$

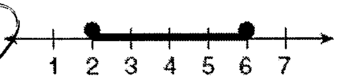
$\frac{2x}{2} < \frac{10}{2}$

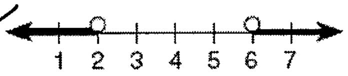
$x<5$

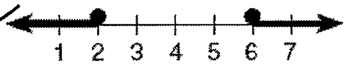
10.

Which graph represents the solution set for $2x - 4 \leq 8$ and $x + 5 \geq 7$?

~~(1)~~


[✓ (2)]


~~(3)~~


~~(4)~~


$2x \leq 12$

$x \leq 6$

$x \geq 2$

11.

The sum of the ages of the three Romano brothers is 63. If their ages can be represented as consecutive integers, what is the age of the middle brother?

x

$x+1$

$x+2$

$3x+3=63$

$3x=60$

$x=20$

[✓ 21]

12. A ribbon 56 centimeters long is cut into two pieces. One of the pieces is three times longer than the other. Find the lengths, in centimeters, of *both* pieces of ribbon.

X

$3X$

$3X+X=56$

$4X=56$

$\boxed{X=14\text{cm}}$ $\boxed{3X=3(14)=42\text{cm}}$

---

13. A prom ticket at Smith High School is \$120. Tom is going to save money for the ticket by walking his neighbor's dog for \$15 per week. If Tom already has saved \$22, what is the minimum number of weeks Tom must walk the dog to earn enough to pay for the prom ticket?

let $x=$ # of weeks

$22+15X \geq 120$

$\frac{15X}{15} \geq \frac{98}{15}$

$x \geq 6.5\overline{3}$

$\boxed{7\text{ weeks}}$

---

14. The tickets for a dance recital cost \$5.00 for adults and \$2.00 for children. If the total number of tickets sold was 295 and the total amount collected was \$1,220, how many adult tickets were sold? [Only an algebraic solution can receive full credit.]

Let $X=$ adults
$Y=$ children

$5X+2Y=1220$
$-2(X+Y=295)$

$\rightarrow$ $5X+\cancel{2Y}=1220$
$-2X\cancel{-2Y}=-590$

$\frac{3X}{3}=\frac{630}{3}$

$X=210$ Aduts

$210+Y=295$
$-210 \quad -210$

$Y=85$ children

$\boxed{\begin{array}{l}210\text{ Adults}\\85\text{ children}\end{array}}$

---

15. Sara's telephone service costs \$21 per month plus \$0.25 for each local call, and long-distance calls are extra. Last month, Sara's bill was \$36.64, and it included \$6.14 in long-distance charges. How many local calls did she make?

$21+.25X+6.14=36.64$

$.25X+27.14=36.64$

$\boxed{x=38\text{ local calls}}$

16. Peter begins his kindergarten year able to spell 10 words. He is going to learn to spell 2 new words every day.

Write an inequality that can be used to determine how many days, *d*, it takes Peter to be able to spell *at least* 75 words.

let x = # of new words

$10 + 2x \geq 75$

Use this inequality to determine the minimum number of whole days it will take for him to be able to spell *at least* 75 words.

$x \geq 32.5$

33 days

17.

Rhonda has \$1.35 in nickels and dimes in her pocket. If she has six more dimes than nickels, which equation can be used to determine x, the number of nickels she has?

(1) $0.05(x + 6) + 0.10x = 1.35$

[✓ (2)] $0.05x + 0.10(x + 6) = 1.35$

(3) $0.05 + 0.10(6x) = 1.35$

(4) $0.15(x + 6) = 1.35$

let

x = # nickels

$x + 6$ = # dimes

$.05x + .10(x+6) = 1.35$

REVIEW OF POLYNOMIALS, EXPONENTS & SCIENTIFIC NOTATION

Exponent Rules: The coefcients always perform the operation in the problem, the exponents never do.

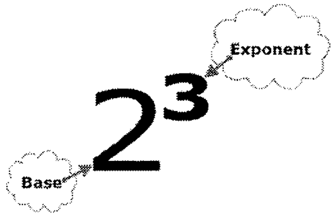
Laws of Exponents

Multiplying Problems:
Multiply coefficients
Add Exponents
Ex: $6x^5 \cdot 2x^2 = 12x^7$

Dividing Problems:
Divide coefficients
Subtract Exponents
Ex: $6x^5 \div 2x^2 = 3x^3$

Adding/Subtracting Problems:
Add/Subtract coefficients
Exponents Stay the Same
Ex: $6x^5 + 2x^5 = 8x^5$

Zero and Negative Exponents:



$$x^0 = 1$$

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{1}{x^{-n}} = x^n$$

Power Rule (Power Raised to Another Power):
Multiply the exponents
Ex: $(3x^2y^3)^3 = 3^3x^6y^9$
$\mathbf{27x^6y^9}$

⚠ **Notice the Difference:** $-3^2 \neq (-3)^2$ ***because*** $\mathbf{-3^2 = -9}$ ***yet*** $(-3)^2 = \mathbf{+9}$

Adding Polynomials:

- Combine like terms
- Like terms have same exponent and same variable
- Add Coefficients, DON'T TOUCH THE EXPONENTS

Ex: $(2x^2 - 3x + 4) + (-5x^2 - 4x + 10) = \mathbf{-3x^2 - 7x + 14}$

Subtracting Polynomials:

- Distribute the negative 1, combine like terms

Ex: $(5x^2+3x-6)-(3x^2-x+5)$

This guy **distributes into these!**

$= 5x^2+3x-6-3x^2+x-5$

$=(5x^2-3x^2)+(3x+x)+(-6-5)$

$= 2x^2+4x-11$

"Subtract/From" Problems:
The "from" expression goes first followed by a subtraction symbol and then the "subtract" expression in parentheses.

Multiplying Polynomials: Each term in the first () multiples each term in the 2nd ().

Ex: $3x^2(x+5)$

$3x^2(x+5) = 3x^2(x)+3x^2(5)$

$= 3x^2x^1+3\cdot5x^2$

$= 3x^3+15x^2$

Ex: $(3g-3)(2g^2+4g-4)$

$6g^3 + 12g^2 - 12g - 6g^2 - 12g + 12$

$\mathbf{6g^3 + 6g^2 - 24g + 12}$

Dividing a Polynomial by a Monomial:

1. Divide **each** term of the polynomial by the monomial
2. Apply your rules for dividing a monomial by a monomial

$$\frac{5xy^2 - 12x^3y^4 - 6}{3x}$$

$$= \frac{5xy^2}{3x} - \frac{12x^3y^4}{3x} - \frac{6}{3x}$$

$$= \frac{5}{3}y^2 - 4x^2y^4 - \frac{2}{x}$$

Standard Form: Terms are written by descending degree.

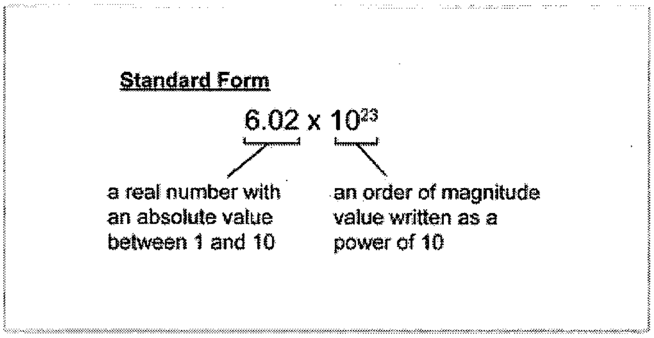
Names of Polynomials by their degree

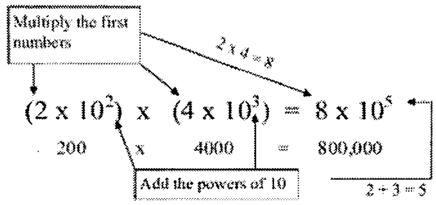
Degree	Name	Example
0	constant	5
1	linear	$2x$
2	quadratic	$x^2 + 3x + 2$
3	cubic	$2x^3 - x^2 + 3x + 2$
4	quartic	$x^4 + 2x^3 - x^2 + 3x + 1$
5	5th degree	$x^5 - 3x^4 + x^3 - 2x^2 + x + 5$
6	sixth degree	$2x^6 + 4x^5 - x^4 + 3x^3 + 2x^2 - x - 1$

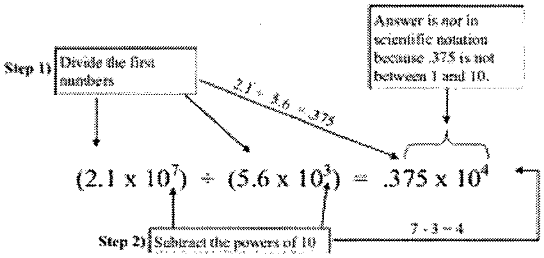
Names of polynomials by the number of terms

Number of terms	Name	Examples
1	monomial	2 $3x$ $-3x^4$
2	binomial	$x + 3$ $x^3 - x^2$ $x + y$
3	trinomial	$x^2 + 3x + 2$ $x^4 - x^2 + 1$ $x^2 + 2xy + y^2$
4 or more	polynomial	$2x^3 - x^2 + 3x + 2$ $x^5 - 2x^2 + x + 5$ $x^2 + 2xy + y^2 + 1$

Scientific Notation: is a way to express very small or very large numbers.





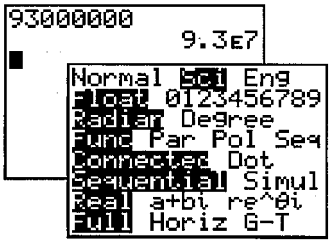


$= 3.75 \text{ x } 10^3$

To express answers in scientific notation:

Press **MODE**, choose **SCI** (scientific notation mode)

Note: The calculator is using the notation 9.3 E 7 to mean 9.3 x 10^7



```
93000000
            9.3E7
```

To insert the E into a calculation, press 2nd *(comma)*, The face of the calculator reads EE above the comma key, but only one E will appear on the screen for your scientific notation. Can be used in either Normal or Sci mode.

Regents Review

1.

When $-9x^5$ is divided by $-3x^3$, $x \neq 0$, the quotient is

(1) $-3x^2$ (3) $-27x^{15}$

[✓ (2) $3x^2$] (4) $27x^8$

$\frac{-9x^5}{-3x^3} =$

$3x^2$

2.

What is the product of $\frac{1}{3}x^2y$ and $\frac{1}{6}xy^3$?

(1) $\frac{1}{2}x^2y^3$ (3) $\frac{1}{18}x^2y^3$

(2) $\frac{1}{9}x^3y^4$ [✓ (4) $\frac{1}{18}x^3y^4$]

$\frac{1}{18}x^3y^4$

3.

Which expression represents $\frac{(2x^3)(8x^5)}{4x^6}$ in simplest form?

(1) x^2 [✓ (3) $4x^2$]

(2) x^9 (4) $4x^9$

$\frac{16x^8}{4x^6} = 4x^2$

4.

Which expression is equivalent to $(3x^2)^3$?

(1) $9x^5$ (3) $27x^5$

(2) $9x^6$ [✓ (4) $27x^6$]

3^3x^6

$27x^6$

5.

What is the value of 2^{-3}?

(1) $\frac{1}{6}$ (3) -6

[✓ (2) $\frac{1}{8}$] (4) -8

$\frac{1}{2^3}$

6.

When $3a^2 - 7a + 6$ is subtracted from $4a^2 - 3a + 4$, the result is

[✓ (1) $a^2 + 4a - 2$] (3) $-a^2 - 4a + 2$

(2) $a^2 - 10a - 2$ (4) $7a^2 - 10a + 10$

$4a^2-3a+4-(3a^2-7a+6)$

$4a^2-3a+4-3a^2+7a-6$

a^2+4a-2

7.

The expression $\frac{5x^6y^2}{x^8y}$ is equivalent to

(1) $5x^2y$ (3) $5x^{14}y^3$

[✓ (2) $\frac{5y}{x^2}$] (4) $\frac{5y^3}{x^{14}}$

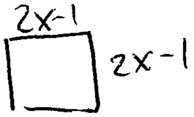
$\frac{5y}{x^2}$

8.

The length of a side of a square window in Jessica's bedroom is represented by $2x - 1$. Which expression represents the area of the window?

(1) $2x^2 + 1$ (3) $4x^2 + 4x - 1$

(2) $4x^2 + 1$ [✓ (4)] $4x^2 - 4x + 1$



$(2x-1)(2x-1)$

$4x^2-2x-2x+1$

$4x^2-4x+1$

9.

What is the product of $-3x^2y$ and $(5xy^2 + xy)$?

[✓ (1)] $-15x^3y^3 - 3x^3y^2$ (3) $-15x^2y^2 - 3x^2y$

(2) $-15x^3y^3 - 3x^3y$ (4) $-15x^3y^3 + xy$

$-3x^2y(5xy^2+xy)$

$-15x^3y^3 - 3x^3y^2$

10.

The sum of $3x^2 + 4x - 2$ and $x^2 - 5x + 3$ is

(1) $4x^2 + x - 1$ (3) $4x^2 + x + 1$

[✓ (2)] $4x^2 - x + 1$ (4) $4x^2 - x - 1$

$4x^2-1x+1$

11.

What is the quotient of 8.05×10^6 and 3.5×10^2?

(1) 2.3×10^3 (3) 2.3×10^8

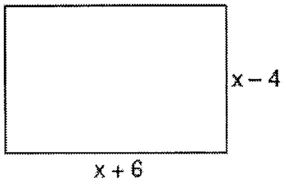
[✓ (2)] 2.3×10^4 (4) 2.3×10^{12}

Show on calculator

2.3×10^4

12.

Express both the perimeter and the area of the rectangle shown in the accompanying diagram as polynomials in simplest form.



$2(x-4)+2(x+6)$

$2x-8+2x+12$

$\boxed{\text{Perim} = 4x+4}$

$(x+6)(x-4)$

$x^2-4x+6x-24$

Area $\boxed{x^2+2x-24}$

13.

Which expression is equivalent to $x^{-1} \cdot y^2$?

(1) xy^2 (3) $\frac{x}{y^2}$

[✓ (2) $\frac{y^2}{x}$] (4) xy^{-2}

$\frac{y^2}{x}$

14.

The expression $\frac{9x^4 - 27x^6}{3x^3}$ is equivalent to

(1) $3x(1 - 3x)$ (3) $3x(1 - 9x^5)$

[✓ (2) $3x(1 - 3x^2)$] (4) $9x^3(1 - x)$

$3x - 9x^3$

$3x(1 - 3x^2)$

15.

The expression $2x^2 - x^2$ is equivalent to

(1) x^0 [✓ (3) x^2]

(2) 2 (4) $-2x^4$

x^2

16.

The expression $(3c)^{-2}$ is equivalent to

(1) $-6c^2$ [✓ (3) $\frac{1}{9c^2}$]

(2) $\frac{1}{3c^2}$ (4) $\frac{3}{c^2}$

$\frac{1}{(3c)^2} = \frac{1}{9c^2}$

17.

What is the sum of $2m^2 + \cancel{3m} - 4$ and $m^2 - \cancel{3m} - 2$?

(1) $m^2 - 6$

[✓ (2) $3m^2 - 6$]

(3) $3m^2 + 6m - 6$

(4) $m^2 + 6m - 2$

$3m^2 - 6$

18.

The expression $(-4a^3b)^2$ is equivalent to

(1) $-16a^6b^2$
[✓ (2) $16a^6b^2$]
(3) $16a^5b^2$
(4) $8a^6b^2$

$(-4)^2a^6b^2$

$16a^6b^2$

19.

~~When $3a^2 - 7a + 6$ is subtracted from $4a^2 - 3a + 4$, the result is~~

~~(1) $a^2 + 4a - 2$~~
~~(2) $a^2 - 10a + 2$~~
~~(3) $-a^2 - 4a + 2$~~
~~(4) $7a^2 - 10a + 10$~~

20.

The expression $(x^2 - 5x - 2) - (-6x^2 - 7x - 3)$ is equivalent to

(1) $7x^2 - 12x - 5$
(2) $7x^2 - 2x + 1$
[✓ (3) $7x^2 + 2x + 1$]
(4) $7x^2 + 2x - 5$

$x^2 - 5x - 2 + 6x^2 + 7x + 3$

$7x^2 + 2x + 1$

21.

If $x \neq 0$, then $\frac{(x^2)^3}{x^5} \bullet 1000$ is equivalent to

[✓ (1) $1000x$]
(2) $1000 + x$
(3) 1000
(4) 0

$\frac{x^6}{x^5} \cdot 1000$

$x \cdot 1000 = 1000x$

22.

What is the sum of $x^2 - 3x + 7$ and $3x^2 + 5x - 9$?

(1) $4x^2 - 8x + 2$
(2) $4x^2 + 2x + 16$
(3) $4x^2 - 2x - 2$
[✓ (4) $4x^2 + 2x - 2$]

$4x^2 + 2x - 2$

23.

The expression $8^{-4} \bullet 8^6$ is equivalent to

(1) 8^{-24}
(2) 8^{-2}
[✓ (3) 8^2]
(4) 8^{10}

24.

Which trinomial is equivalent to $(3x - 1)(x + 4)$?

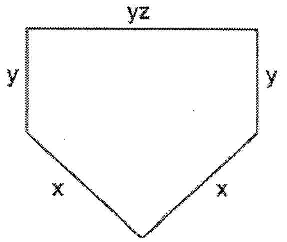
[✓ (1) $3x^2 + 11x - 4$]
(2) $3x^2 + 13x - 4$
(3) $3x^2 - 11x + 4$
(4) $3x^2 + 11x + 4$

$3x^2 + 12x - 1x - 4$

$3x^2 + 11x - 4$

25.

The lengths of the sides of home plate in a baseball field are represented by the expressions in the accompanying figure.



$2x+2y+yz$

Which expression represents the perimeter of the figure?

(1) $5xyz$ (3) $2x + 3yz$

(2) $x^2 + y^3z$ [✓ (4) $2x + 2y + yz$]

26.

The length of a rectangular window is 5 feet more than its width, w. The area of the window is 36 square feet. Which equation could be used to find the dimensions of the window?

[A] $w^2-5w-36=0$ [B] $w^2-5w+36=0$

[C] $w^2+5w+36=0$ [✓ [D] $w^2+5w-36=0$]

$w(w+5)=36$

$w^2+5w=36$

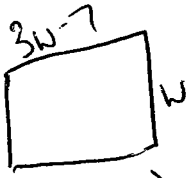
$w^2+5w-36=0$

27.

The length of a rectangular room is 7 less than three times the width, w, of the room. Which expression represents the area of the room?

[A] $3w-4$ [B] $3w^2-4w$

[✓ [C] $3w^2-7w$] [D] $3w-7$



$w(3w-7)=3w^2-7w$

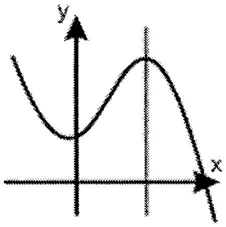
REVIEW OF FUNCTIONS

To determine if a relation is a function:

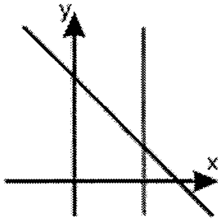
Graphs: must pass the vertical line test (vertical line can never intersect the graph more than once)

Points: All x-values must be different in a function

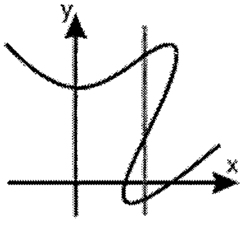
Function: *A function is a set of ordered pairs in which each x-element has only ONE y-element associated with it.*



This is a function



This is a function



This is NOT a function

Ex: (1,2), (2, 4), (4,5), (6,7)
This is a function

Ex: (1,2), (1,4), (5, 8), (8, 3)
This is NOT a function

f(x) notation: If f(x) is given and we want to find "f(some number)", we substitute the number in place of x on the right side of the equation.

Ex: A function is represented by $f(x) = 2x + 5$. Find $f(3)$.

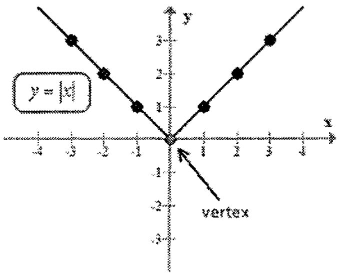
To find $f(3)$, replace the x-value with 3. $f(\mathbf{3}) = 2(\mathbf{3}) + 5 = 11$.

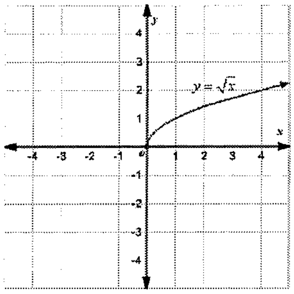
Domain: a list of the x-values
Range: a list of y-values

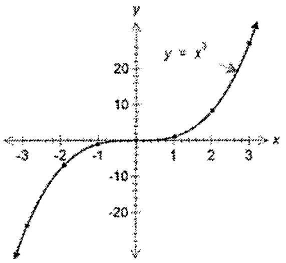
Restricted Domains - many functions have a domain of "all Real numbers", EXCEPT…

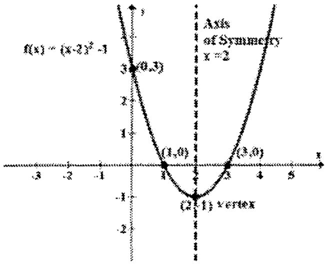
1) **Fractions:** what ever number(s) make the fraction UNDEFINED are NOT in the domain. Ex: Domain of $\frac{2x+1}{10-X}$ $is\ x \neq 10$
2) **Square Roots:** the radicand must be ≥ zero.
Ex: Domain of $\sqrt{2x-5}$ is $x \geq \frac{5}{2}$
3) **Square Roots in Denominator:** the radicand must be > zero.
Ex. Domain of $\frac{3x+7}{\sqrt{2x-5}}$ is $x > \frac{5}{2}$

Types of graphs







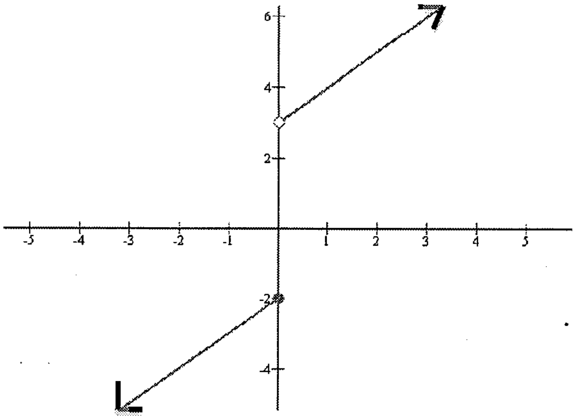


Piecewise Functions

A piecewise function is called piecewise because it acts differently on different "pieces" of the number line. For example, consider this function:

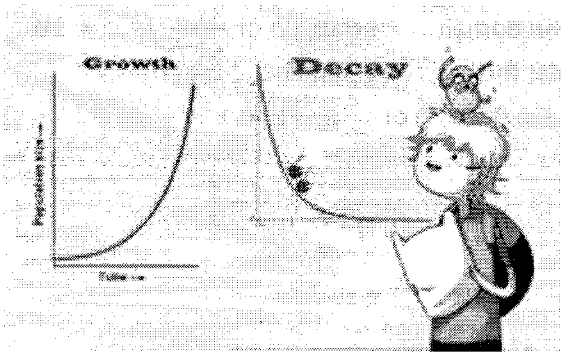
$$f(x)=\begin{cases} x-2, & x\leq 0 \\ x+3, & x>0 \end{cases}$$

This function has two parts. For all values of x that are 0 or less, we use the line y = x – 2. We stop at the point (0, -2) since for x-values greater than 0, we use a different line. For values of x that are strictly greater than 0, we use the line y = x+3. Here's the graph of this function:



Graph of Exponential Function: $y = a * b^x$

- when *a* > 0 and the *b* is between 0 and 1, the graph will be decreasing (decaying).
- when *a* > 0 and the *b* is greater than 1, the graph will be increasing (growing).



There are two functions that can be easily used to illustrate the concepts of growth and decay in applied situations. When a quantity grows by a fixed percent at regular intervals, the pattern can be represented by the functions,

Growth:	**Decay:**
$y = a(1+r)^x$	$y = a(1-r)^x$

a = initial **amount** before measuring growth/decay
r = growth/decay **rate** (must convert to a decimal)
x = number of **time (years)** intervals that have passed

Ex: A bank is advertising that new customers can open a savings account with a $3\frac{3}{4}\%$ interest rate compounded annually. Robert invests \$5,000 in an account at this rate. If he makes no additional deposits or withdrawals on his account, find the amount of money he will have, to the nearest cent, after three years.

$$y = 5000(1 + 0.0375)^3$$
$$y = 5000(1.0375)^3$$
$$y = \$5583.86$$

Compound Interest (w/ Months): Solving Word Problems Using the Compound Interest Model

You deposit \$1500 in a bank account that pays 4% annual interest. Find the balance after 6 years if the interest is compounded monthly. Round to the nearest cent.

$$y = \underset{\substack{\uparrow \\ \text{starting} \\ \text{value}}}{a} \cdot \left(1 + \frac{\overset{\text{interest rate}}{\overset{\searrow}{r}}}{\underset{\substack{\uparrow \\ \text{compounding} \\ \text{rate}}}{n}}\right)^{nx} \nwarrow \text{\# of intervals}$$

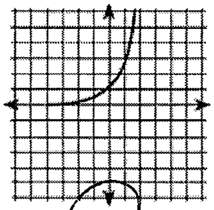
~~y = 1500~~ ⟵ Fix

$$y = 1500\left(1 + \frac{.04}{12}\right)^{12(6)}$$

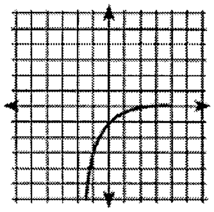
$$\boxed{y = \$1906.11}$$

Regents Review

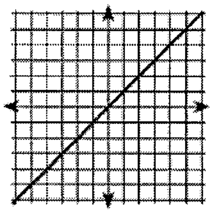
1. Which graph is a correct representation of the function $f(x) = 3^x$?



[✓ (1)]



(2)



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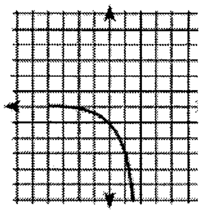


(4)

---

2. The graph of the equation $y = 3^x$ contains which point?

(1) $(1,9)$ [✓ (2)] $\left(-2,\frac{1}{9}\right)$ (3) $(2,6)$ (4) $\left(-3,\frac{1}{9}\right)$

$0<b<1$ decay

$b>1$ growth

---

3. Which function represents exponential decay?

[✓ (1)] $f(x)=100(.9)^x$ ~~(2)~~ $f(x)=10(1.09)^x$ ~~(3)~~ $f(x)=1.9^x$ ~~(4)~~ $f(x)=\frac{1}{2}(9)^x$

---

4. A car depreciates (loses value) at a rate of $\boxed{4.5\%}$ annually. Greg purchased a car for \$12,500. Which equation can be used to determine the value of the car, V, after 5 years?

(1) $V=12{,}500(0.55)^5$ [✓ (2)] $V=12{,}500(0.955)^5$ (3) $V=12{,}500(1.045)^5$ (4) $V=12{,}500(1.45)^5$

$(1-.045)$

$.955$

---

5. A bank is advertising that new customers can open a savings account with a $3\frac{1}{2}\%$ interest rate compounded annually. Robert invests \$5000 in an account at this rate. If he makes no additional deposits or withdrawals on his account, find the amount of money he will have, to the nearest cent, after three years.

$5000(1+.035)^3$

$\boxed{\$5543.59}$

---

6. Joseph conducted a science experiment involving the growth of bacteria. He measured the number of bacteria hourly for 6 hours. The data is summarized in the accompanying table. What type of regression would best fit the data?

~~(1)~~ Linear [✓ (2)] Exponential (3) Quadratic ~~(4)~~ Absolute Value

Hour	Number of Bacteria	
0	300	170
1	470	255
2	725	425
3	1150	625
4	1800	950
5	2750	1650
6	4400	

---

7. Is the equation $A = 21000(1-0.12)^t$ a model of exponential growth or exponential decay, and what is the rate (percent) of change per time period?

~~(1)~~ exponential growth and 12%　　[✓ (3) exponential decay and 12%]
~~(2)~~ exponential growth and 88%　　(4) exponential decay and 88%

---

8. Mr. Smith invested \$2500 in a savings account that earns 3% interest compounded annually. He made no additional deposits or withdrawals. Which expression can be used to determine the number of dollars in this account at the end of 4 years?

[✓ (1) $2500(1+0.03)^4$]　(2) $2500(1+0.3)^4$　(3) $2500(\cancel{1+}0.04)^4$　(4) $2500(\cancel{1+}0.4)^4$

3% = .03

---

9. Daniel's Print Shop purchased a new printer for \$35,000. Each year it depreciates (loses value) at a rate of 5%. What will its approximate value be at the end of the fourth year?

1) \$33,250.00　(2) \$30,008.13　[✓ (3) \$28,507.72]　(4) \$27,082.33

$35000(1-.05)^4$

---

10. Which relation is *not* a function?

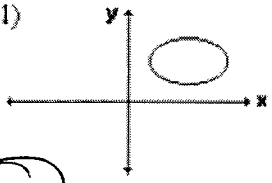
✓ 1, 2, 3, 4　(1) $\{(1,5),(2,6),(3,6),(4,7)\}$　　[✓ (3) $\{(-1,6),(1,3),(2,5),(1,7)\}$] -1, 1, 2, 1

✓ 3, 2, -3, 4　(2) $\{(3,4),(2,1),(-3,6),(4,7)\}$　　(4) $\{(-1,2),(5,0),(0,5),(2,-1)\}$

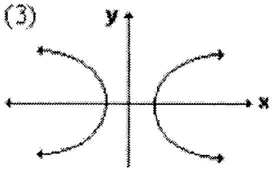
---

11. Which graph of a relation is also a function?

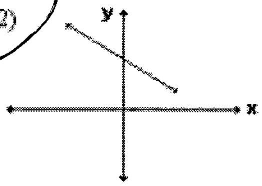
(1)



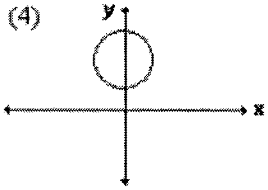
(3)



[✓ (2)]



(4)



---

12. Miller Brothers Plumbing Inc. charges by the hour or part of an hour as follows for repairs:

$$\text{Total charge for repairs } \begin{cases} \$75 & \text{if } 0 < h \le 1 \\ \$75 + 50(h-1) & \text{if } h > 1 \end{cases}$$

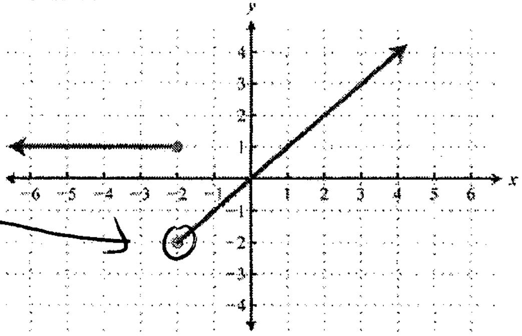
If h represents the number of hours worked, what is the charge for a 4 hour repair?

(1) \$75　(2) \$125　(3) \$175　[✓ (4) \$225]

75 + 50(4-1)
75 + 50(3)
75 + 150

13. What is the rule for the piecewise function shown below?

Fix



[✓ (1)] $\begin{cases} 1 & \text{if } x \le -2 \\ x & \text{if } x > -2 \end{cases}$ (2) $\begin{cases} 1 & \text{if } x < -2 \\ x & \text{if } x \ge -2 \end{cases}$

Using the graph left find the values of the function given the domain values (x) indicated

$f(-4) =$ 1

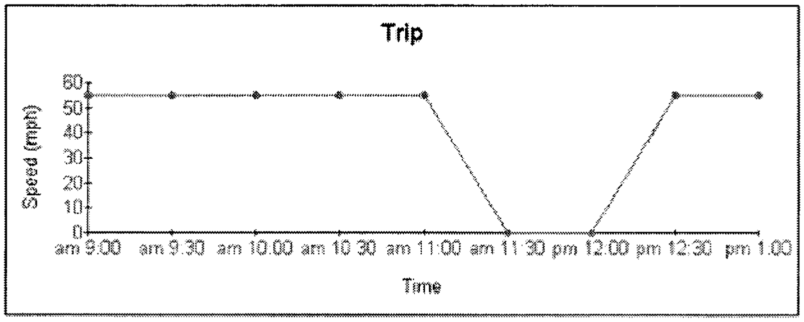
$f(-2) =$ 1

$f(0) =$ 0

$f(3) =$ 3

outputs

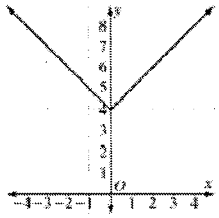
15. Sharon and John are making a line graph of their trip to New Orleans. They plotted their speed every half hour. What do you think happened from 11:30 to 12:00?



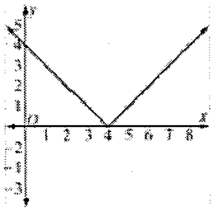
A. They reached their destination.
[✓ B.] They left the highway to stop for lunch.
C. They increased their speed.
D. They decreased their speed.

16. Which of the following is the graph of $y = |x| - 4$?

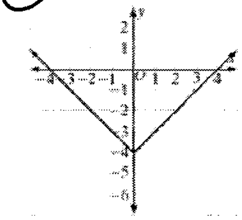
(1)



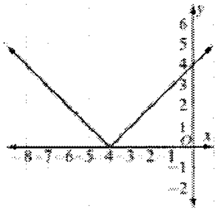
(2)



[✓ (3)]



(4)



18. $f(x)=\begin{cases} x+5 & x<-2 \\ x^2+2x+3 & x\ge -2 \end{cases}$

$x<-2$

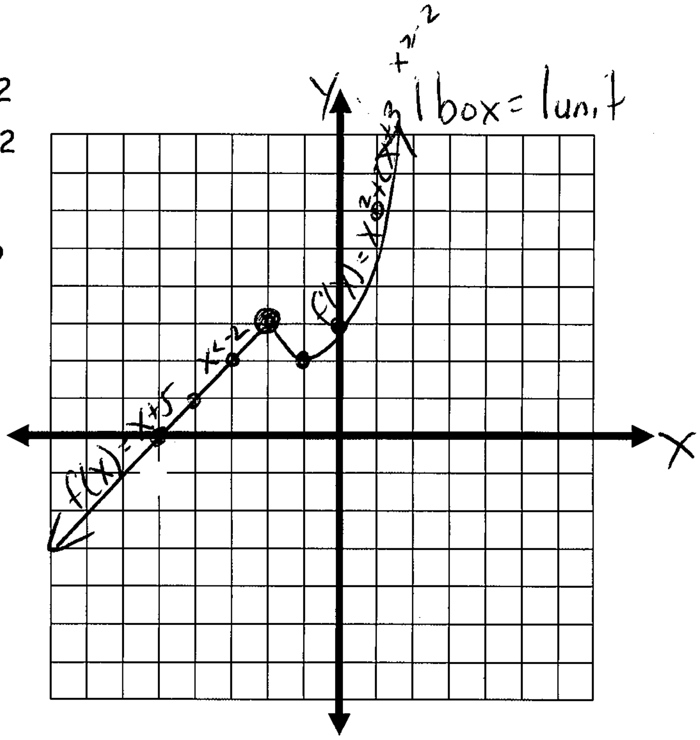
$y=x+5$

X	Y
* -2	3
-3	2
-4	1
-5	0

$x\ge -2$

$y=x^2+2x+3$

X	Y
-2	3
-1	2
0	3
1	6
2	11



19. $f(x)=\begin{cases} 2x+1 & x\ge 1 \\ x^2+3 & x<1 \end{cases}$

$x\ge 1$

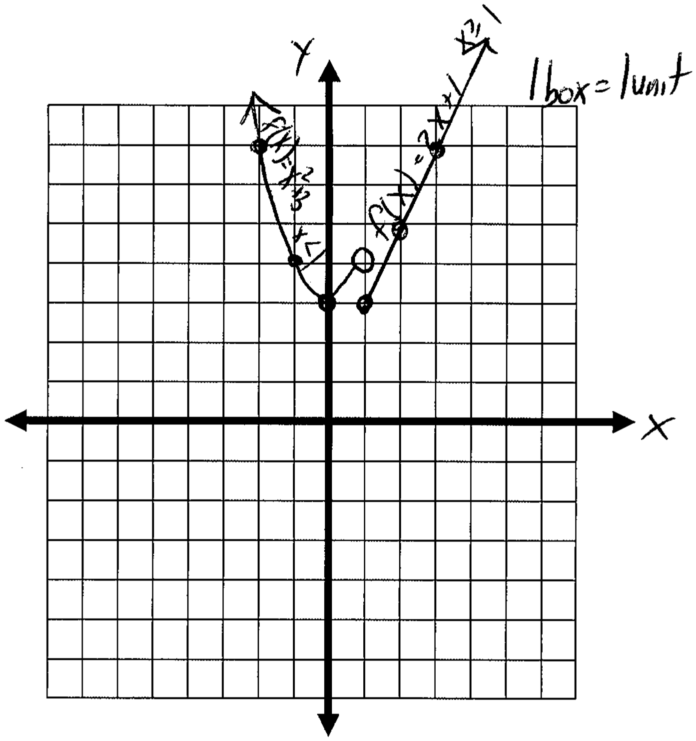
$f(x)=2x+1$

X	Y
1	3
2	5
3	7
4	9
5	11

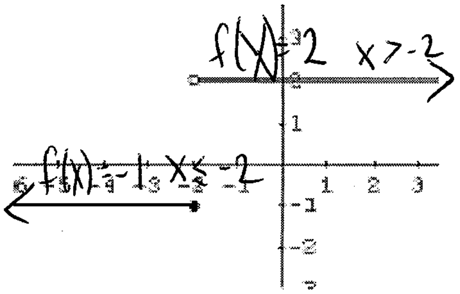
$x<1$

$f(x)=x^2+3$

X	Y
* 1	4
0	3
-1	4
-2	7



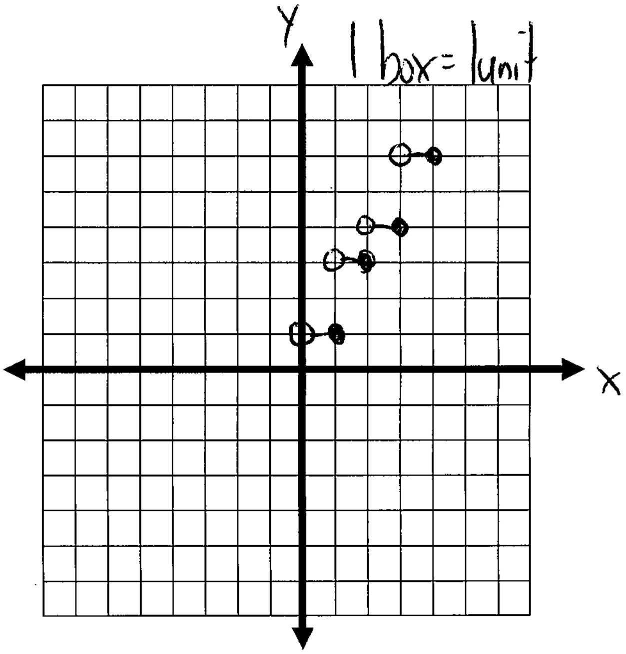
20. **Write the function rule.**



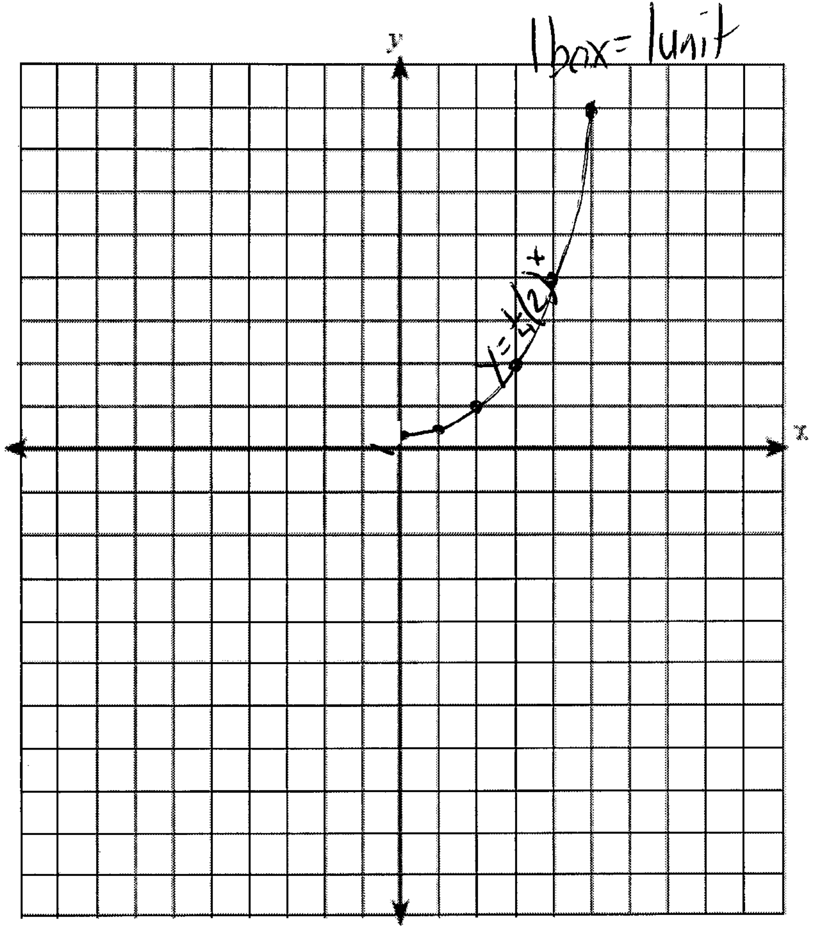
$$f(x) = \begin{cases} 2 & x>-2 \\ -1 & x\leq -2 \end{cases}$$

Step function

$$21.\ f(x) = \begin{cases} 1, & \text{if} \quad 0 < x \leq 1 \\ 3, & \text{if} \quad 1 < x \leq 2 \\ 4, & \text{if} \quad 2 < x \leq 3 \\ 6, & \text{if} \quad 3 < x \leq 4 \end{cases}$$



22. a) Sketch the graph of all of the solutions to the equation $y=\frac{1}{4}(2)^x$, where $0 \le x \le 5$.



X	Y
0	.25
1	.5
2	1
3	2
4	4
5	8

b) Find the average rate of change between $f(2)$ and $f(5)$.

$f(2)=1$

$f(5)=8$

$\boxed{\frac{7}{3}}$

REVIEW OF LINEAR EQUATIONS

$$\text{Slope} = \frac{Vertical\ change}{Horizontal\ change} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-Intercept Form	Point-Slope Form
Use this form when you know the **slope** and the **y-intercept** (where the line crosses the y-axis). $y = mx + b$ m = slope b = y-intercept	Use this form when you know a **point** on the line and the **slope** (or can determine the slope). $y - y_1 = m(x - x_1)$ m = slope (x_1, y_1) = any point on the line

Horizontal Lines	Vertical Lines
$y = 3$ (or any number) Lines that are horizontal have a slope of zero. ↔	↕ $x = -2$ (or any number) Lines that are vertical have no slope (slope is undefined).

There is a way to "fake" the calculator into producing a vertical line.
To graph the vertical line ***x* = -2**:
Y1 = A Big Number (*x* +2)
where "A Big Number" is around 1,000,000.

Writing the equation of a line:

Step 1: Find the slope (m)

Step 2: Find the y-intercept by plugging the slope (m) and point (x,y) into y=mx + b to solve for "b".

Step 3: Write the equation of the line.

Given	Need	Process	
1 point & a parallel line *(3,2) parallel to:* $y = -4x + 1$ Knowing the properties of parallel lines we know the slope is the same for both lines. So the slope is -4	y-intercept	Find y-intercept $y = mx + b$ $(2) = (-4)(3) + b$ $2 = -12 + b$ $+12 \quad +12$ $14 = b$	Plug in the slope & y-intercept to create the new equation. $m = -4 \quad b = 12$ $y = mx + b$ $y = (-4)x + (14)$ $\boxed{y = -4x + 14}$
1 point & a perpendicular line *(8,3) perpendicular to:* $y = -4x + 1$ Knowing the properties of perpendicular lines we know the slope of the lines are opposite reciprocals of each other. So the slope is $\frac{1}{4}$	y-intercept	Find y-intercept $y = mx + b$ $(3) = \left(\frac{1}{4}\right)(8) + b$ $3 = 2 + b$ $-2 \quad -2$ $1 = b$	Plug in the slope & y-intercept to create the new equation. $m = \frac{1}{4} \quad b = 1$ $y = mx + b$ $y = \left(\frac{1}{4}\right)x + (1)$ $\boxed{y = \frac{1}{4}x + 1}$

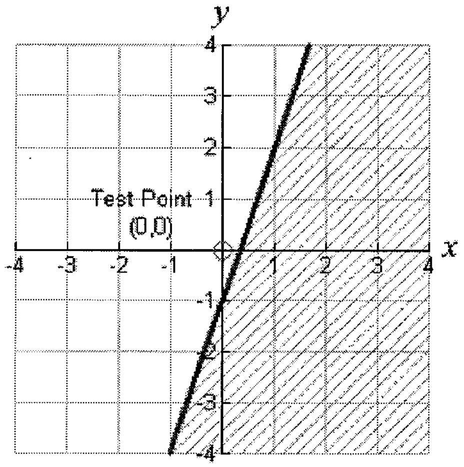
REVIEW OF GRAPHING LINEAR INEQUALITIES

If you can graph a straight line, you can graph an inequality!!!!

Graphing an Inequality

1. Solve the equation for **y** (if necessary).
2. Graph the equation as if it contained an = sign.
3. Draw the line solid if the inequality is $\leq$ or $\geq$
4. Draw the line dashed if the inequality is < or >
5. Pick a point **not** on the line to use as a test point. The point (0,0) is a good test point if it is not on the line.
6. If the point makes the inequality true, shade that side of the line. If the point does not make the inequality true, shade the opposite side of the line.

Quick Guide:
< Dotted, shade below
> Dotted, shade above
≤ Solid, shade below
≥ Solid, shade above



Graph the inequality
$y \leq 3x - 1$

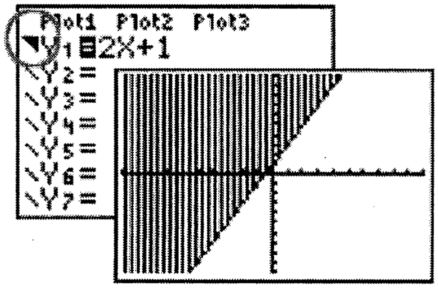
1. Graph the line $y = 3x - 1$.
2. Pick a test point. (0,0) was used.
3. The test point is false in the inequality

$0 \leq 3(0) - 1$

$0 \leq -1$ **false**

4. Since the test was false, do not shade OVER the point (0,0) -- shade the opposite side of the line.
5. The line, itself, is SOLID because this problem is "less than or EQUAL TO."

Example 1: Graph $y \geq 2x + 1$



- Enter $2x + 1$ into Y_1
- Arrow to the far left side of Y_1
- Hit ENTER until the "shade above" symbol is displayed.
- Hit **ZOOM #6 ZStandard** (for a 10x10 window)
- **Graph**
- NOTE: You will have to determine whether to draw a solid line or a dotted line for $y = 2x + 1$. This problem uses a solid line because of the "less than or equal to" sign. The calculator will display a solid line at all times.

REVIEW OF SYSTEMS – LINEAR EQUATIONS

Graphically:

- If you can graph a straight line, you can solve systems of equations graphically!
- The process is very easy. Simply graph the two lines and look for the point where they intersect (cross).

Solve graphically:

$$4x - 6y = 12$$
$$2x + 2y = 6$$

First, solve each equation for "$y =$".

$4x - 6y = 12$

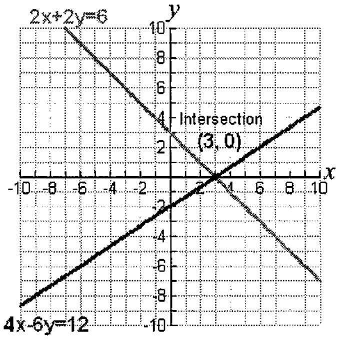
$$4x = 6y + 12$$
$$4x - 12 = 6y$$
$$6y = 4x - 12$$
$$y = \frac{4x}{6} - \frac{12}{6}$$
$$y = \frac{2}{3}x - 2$$

slope = $\frac{2}{3}$
***y*-intercept = -2**

$2x + 2y = 6$

$$2x + 2y = 6$$
$$2y = -2x + 6$$
$$y = \frac{-2x}{2} + \frac{6}{2}$$
$$y = -x + 3$$

slope = -1
***y*-intercept = 3**

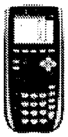


Graph the lines.

The slope intercept method of graphing was used in this example.

The point of intersection of the two lines, (3,0), is the solution to the system of equations.

This means that (3,0), when substituted into either equation, will make them both true. Don't forget to CHECK!!!



1. Enter the first equation into **Y_1**.
2. Enter the second equation into **Y_2**.
3. Hit **GRAPH.**

4. Use the **INTERSECT** option to find where the two graphs intersect (the answer).
   2nd TRACE (CALC) #5 intersect
   Move spider close to the intersection.
   Hit **ENTER** 3 times.

Algebraically using SUBSTITUTION:

- **Substitution (Steps)**
  1. **Substitute one equation into the other equation for one of the variables.**
  2. **Solve for that variable**
  3. **Substitute answer into the either equation to find the value of the remaining variable.**
  4. **Check your solutions in both equations.**

Example

$2x + y = 19$

$y = x + 1$

$2x + \quad y \quad = 19$

step 1. $2x +(x + 1) = 19$

step 2. $3x + 1 = 19$

$\underline{\quad -1 \quad -1}$

$3x \; = 18$

$x = 6$

step 3. $y = 6 + 1$

$y = 7$

ck: $2x + y = 19$

$2(6) + 7 = 19$

$12 + 7 = 19$

$19 =19$

$y = x + 1$

$7 = 6 + 1$

$7 = 7$

Algebraically using ELIMINATION:

Elimination (Steps)

1. **Choose to eliminate one of the variables**
2. **Make the coefficients the same with different signs for the variable you wish to eliminate.**
3. **Add vertically to solve for the remaining variable.**
4. **Use the solution to substitute into the other equation.**
5. **Solve for remaining variable.**
6. **Check your solutions in both equations.**

Example

$3x + 3y = 24$

$5x + y = 12$

step 1: $3x + 3y = 24$ $\qquad 3x + 3y = 24$

step 2: $\underline{-3(5x + y = 12)}$ $\qquad \underline{-15x - 3y = -36}$

step 3: $\qquad -12x \quad = -12$

$x = 1$

step 4: $5x + y = 12$

$5(1) + y = 12$

$5 + y = 12$

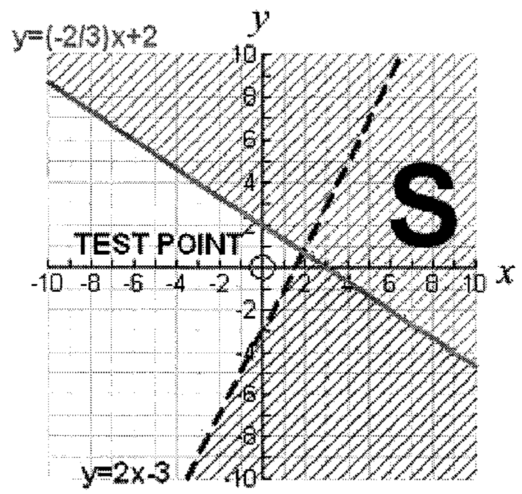
$y = 7$

REVIEW OF SYSTEMS - INEQUALITIES

Two Inequalities: Graph lines on the coordinate plane then shade according to the appropriate rules below. The solution set is the region on the graph that was shaded by both inequalities. Label it "S".

Step 1: Rewrite the inequality in y = mx + b form (using the appropriate inequality sign).
Step 2: Graph the line, using y = mx + b; determining if the inequality has a solid or dashed line (see below).
Step 3: Shade according to the chart below or select a TEST POINT to determine which region to shade. Select any point **not** on the line and substitute the point into the original inequality. If the inequality is true, shade the region where the point is located. If the inequality is false, shade the region opposite the point.

	Solid Line	Dashed line
Shade above the line	$\geq$ "Greater than or equal to"	$>$ "Greater than"
Shade below the line	$\leq$ "Less than or equal to"	$<$ "less than"



Graph the system

$$y < 2x - 3$$

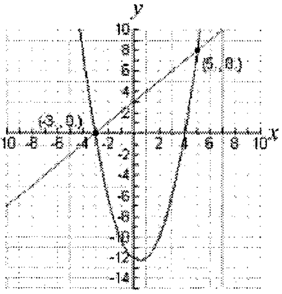
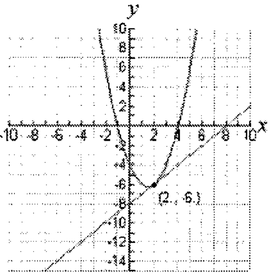
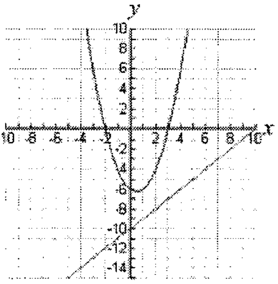
$$y \geq \frac{-2}{3}x + 2$$

Graph each inequality as if it was stated in "y=" form.

The solution, **S**, is where the two shadings overlap one another.

REVIEW OF SYSTEMS – LINEAR/QUADRATIC

In a linear- quadratic system where only one variable in the quadratic is squared, the graphs will be a parabola and a straight line. When graphing a parabola and a straight line on the same set of axes, three situations are possible.

		
The equations will intersect in two locations. Two real solutions.	The equations will intersect in one location. One real solution.	The equations will not intersect. No real solutions.

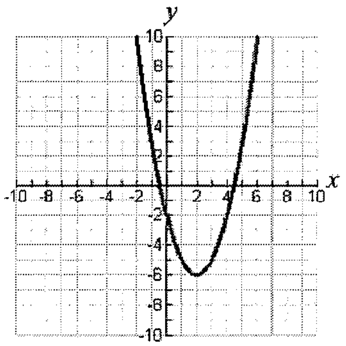
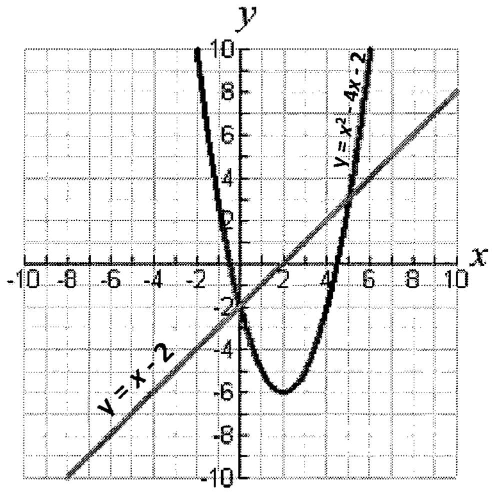
Solving Graphically:

EX: Solve the following system of equations graphically:

$\mathbf{y = x^2 - 4x - 2}$ (quadratic equation of form $y = ax^2 + bx + c$)

$\mathbf{y = x - 2}$ (linear equation of form $y = mx + b$)

<table>
<tr><td colspan="2">Step 1: Graph one of the equations. Let's graph the quadratic equation first. By its form, $y = x^2 - 4x - 2$, we know it is a parabola.</td></tr>
<tr><td>Rather than picking numbers at random to form our table of values, let's find the axis of symmetry where the turning point of the parabola will occur.</td><td>To find the axis of symmetry, we use the formula $\mathbf{x} = {}^{-b}/_{2a}$
In this example, a = 1, b = -4, and c = -2.
Substituting we get:
$x = -(-4)/2(1)$
$x = 4/2$
$x = 2$ axis of symmetry</td></tr>
<tr><td>Since the <i>x</i>-coordinate of the turning point is 2, let's use this value as the middle value for <i>x</i> in our table. We will also include 3 values above and below 2 in our table.</td><td>
<table>
<tr><th><i>x</i></th><th><i>y</i></th></tr>
<tr><td>-1</td><td></td></tr>
<tr><td>0</td><td></td></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>5</td><td></td></tr>
</table>
<table>
<tr><th><i>x</i></th><th><i>y</i></th></tr>
<tr><td>-1</td><td>3</td></tr>
<tr><td>0</td><td>-2</td></tr>
<tr><td>1</td><td>-5</td></tr>
<tr><td>2</td><td>-6</td></tr>
<tr><td>3</td><td>-5</td></tr>
<tr><td>4</td><td>-2</td></tr>
<tr><td>5</td><td>3</td></tr>
</table>
</td></tr>
</table>

Next, graph the points from the table to get the graph of the parabola at the right.	
Step 2: Graph the second equation: Now graph the linear equation, a straight line, *y* = *x* - 2 on the same set of axes.	To graph the straight line we need to know the slope and the y-intercept. Remember, from the form, *y* = m*x* + b, m is the slope and b is the*y*-intercept. For our equation, m = 1, b = -2.
Draw the graph of the line starting at -2 on the *y*-axis. Use slope (which is rise over run) to find other points by going up 1 and to the right 1, or down 1 and to the left 1.	
Step 3: Find the intersection points (where they cross). The last step is to find the point(s) where the two graphs intersect. This is the solution set, the answer, of the system of equations.	Our graphs intersect at 2 points whose coordinates are (0,-2) and (5,3). **The solution set for this problem is:** **{(0,-2),(5,3)}**

Solving Algebraically (substitution): $y = x^2 - x - 6$ (quadratic equation)
$y = 2x - 2$ (linear equation)

<table>
<tr><td>First, we solve for one of the variables in the linear equation.</td><td>$y = 2x - 2$</td><td>Since this is already done for us in this example, we can go to the next step.</td></tr>
<tr><td>Next, we substitute for that variable in the quadratic equation, and solve the resulting equation.</td><td>$y = x^2 - x - 6$
$2x - 2 = x^2 - x - 6$
$2x = x^2 - x - 4$

$0 = x^2 - 3x - 4$

$0 = (x - 4)(x + 1)$

$x - 4 = 0$ $x + 1 = 0$
$x = 4$ $x = -1$</td><td>Add 2 to both sides.
Subtract 2<i>x</i> from both sides.

Factor.

Set each factor = 0 and solve.</td></tr>
<tr><td colspan="3">We now have two values for <i>x</i>, but we still need to find the corresponding values for <i>y</i>.</td></tr>
<tr><td>We find the y-values by substituting each value of <i>x</i> into the linear equation.</td><td><table><tr><th>$y = 2x - 2$</th><th>Check 4</th></tr><tr><td>$y = 2(4) - 2$
$y = 8 - 2$
$y = 6$</td><td>(4, 6)</td></tr></table></td><td><table><tr><th>$y = 2x - 2$</th><th>Check -1</th></tr><tr><td>$y = 2(-1) - 2$
$y = -2 - 2$
$y = -4$</td><td>(-1, -4)</td></tr></table></td></tr>
<tr><td>Now we have 2 possible solutions for the system: (4,6) and (-1,-4). We need to check each solution in each equation.</td><td><table><tr><th>Check#1: (4, 6)</th></tr><tr><td>$y = x^2 - x - 6$

$6 = (4)^2 - 4 - 6$
$6 = 16 - 4 - 6$
$6 = 6$ it checks !

$y = 2x - 2$

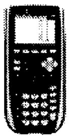
$6 = 2(4) - 2$
$6 = 8 - 2$
$6 = 6$ it also checks !</td></tr></table></td><td><table><tr><th>Check#2: (-1, -4)</th></tr><tr><td>$y = x^2 - x - 6$

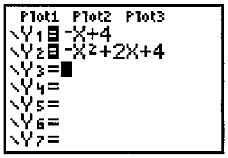
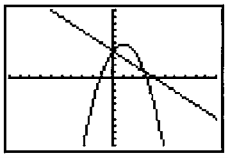
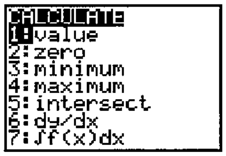
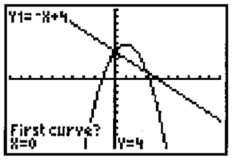
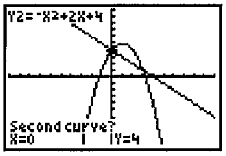
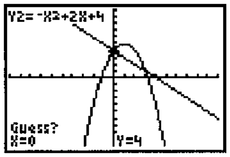
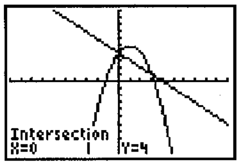
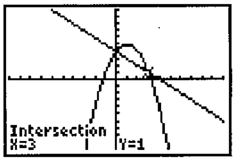
$-4 = (-1)^2 - (-1) - 6$
$-4 = 1 + 1 - 6$
$-4 = -4$ it checks !

$y = 2x - 2$

$-4 = 2(-1) - 2$
$-4 = -2 - 2$
$-4 = -4$ it also checks !</td></tr></table></td></tr>
<tr><td>We finally have our solution set for this linear quadratic system.</td><td>$\boxed{\{(4, 6), (-1, -4)\}}$</td><td></td></tr>
</table>

Solve graphically: $y = -x^2 + 2x + 4$ (*quadratic-parabola*)
$x + y = 4$ (*linear*)



1.	Change the linear equation to "y=" form.	$y = -x + 4$
2.	Enter the equations as "y_1=" and "y_2=". (Be sure to use the negative key, not the subtraction key, for entering negative values.)	 Plot1 Plot2 Plot3 \Y1=-X+4 \Y2=-X²+2X+4 \Y3= \Y4= \Y5= \Y6= \Y7=
3.	Hit GRAPH to see if and where the graphs intersect. (Using **ZOOM #6: ZStandard** creates a 10 x 10 viewing window. You may need to adjust the **WINDOW** to see a clear picture of the intersection locations for the two graphs.)	
4.	Under **CALC** (2nd Trace) choose **#5 intersect** to find the points where the graphs intersect.	
5.	When prompted for the "First curve?", move the spider on, or near, a point of intersection. Hit **Enter.**	 Y1=-X+4 First curve? X=0 Y=4
6.	When prompted for the "Second curve?", just hit **Enter.**	 Y2=-X2+2X+4 Second curve? X=0 Y=4
7.	Ignore the prompt for "Guess?", and hit **Enter.**	 Y2=-X2+2X+4 Guess? X=0 Y=4
8.	Read the answers as to the coordinates of the point of intersection. These coordinates appear at the bottom of the screen. Point of intersection (left side): **(0,4)**	 Intersection X=0 Y=4
9.	If your graphs have a second point of intersection, repeat this process to find the second point. Choose the **#5 intersect** choice and repeat the steps for finding the intersection. Point of intersection (right side): **(3,1)**	 Intersection X=3 Y=1

Solving linear systems algebraically

1\. Substitution

$4x+3y=27$
$y=2x-1$

$4x+3(2x-1)=27$
$4x+6x-3=27$
$10x-3=27$
$10x=30$
$x=3$

$y=2(3)-1$
$y=6-1$
$y=5$

$\{(3,5)\}$

2\. Sub.

$y=x+3$
$3x+2y=26$

$3x+2(x+3)=26$
$3x+2x+6=26$
$5x+6=26$
$5x=20$
$x=4$

$y=4+3$
$y=7$

$\{(4,7)\}$

3\. Elim

$8x+\cancel{5y}=9$
$2x-\cancel{5y}=-4$

$10x=5$
$x=\frac{1}{2}$

$2(\frac{1}{2})-5y=-4$
$1-5y=-4$
$-5y=-5$
$y=+1$

$\{(\frac{1}{2},1)\}$

4\.

$5x+3y=14$
$-3\,(2x+y=6)$

$5x+3y=14$
$-6x-3y=-18$

$-1x=-4$
$x=4$

$5(4)+3y=14$
$20+3y=14$
$3y=-6$
$y=-2$

$\{(4,-2)\}$

5\.

$-2(2x+3y=7)$
$4x-5y=25$

$-4x-6y=-14$
$4x-5y=25$

$-11y=11$
$y=-1$

$4x-5(-1)=25$
$4x+5=25$
$4x=20$
$x=5$

$\{(5,-1)\}$

6\.

$-2(3x+5y=7)$
$3\,(2x+4y=6)$

$-6x-10y=-14$
$6x+12y=18$

$2y=4$
$y=2$

$3x+5(2)=7$
$3x+10=7$
$3x=-3$
$x=-1$

$\{(-1,2)\}$

7\. The sum of two numbers is 36. Their difference is 24. Find the numbers

$x+y=36$
$x-y=24$

$2x=60$

$\boxed{x=30}$

$30-y=24$
$-y=-6$

$\boxed{y=6}$

8. The owner of men's clothing store bought six belts and eight hats for $140. A week later, at the same prices, he bought nine belts and six hats for $132. Find the price of a belt and the price of a hat.

Let
b = # of belts
h = # of halts

$3(6b + 8h = 140)$
$-2(9b + 6h = 132)$

$\cancel{18b} + 24h = 420$
$\cancel{-18b} - 12h = -264$
$12h = 156$
$h = 13$

$6b + 8(13) = 140$
$6b = 36$
$\frac{6b}{6} = \frac{36}{6}$
$b = 6$

$\boxed{\text{13 hats, 6 belts}}$

9. What is the y-intercept of the line whose equation is $y = 6x - 7$?

(1) −6 (2) 6 (3) 7 [✓ (4) −7]

10. Which ordered pair is the solution for the system of equations below?

$2x + y = 18$
$x - y = -6$

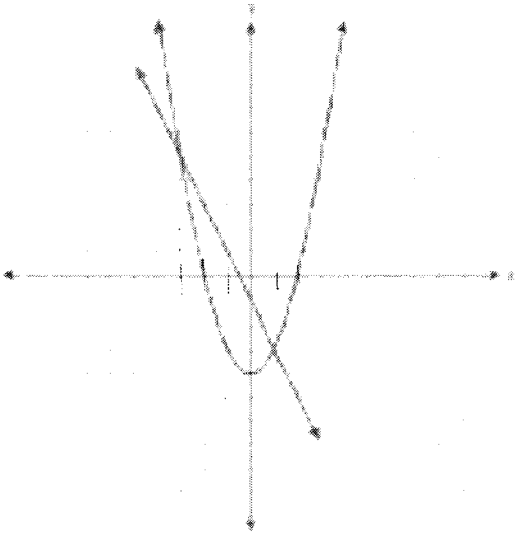
[✓ (1) (4,10)] (2) (4,−10) ~~(3) (8,3)~~ ~~(4) (6,12)~~

$3x = 12$
$x = 4$

$4 - y = -6$
$-y = 10$
$y = 10$

11. Which ordered pair is a solution of the system of equations shown in the graph below?

(1) (−3,1) [✓ (2) (−3,5)] (3) (0,−1) (4) (0,−4)



12. What is the slope of the line that passes through the points (−6,1) and (4,−4)?

(1) −2 (2) 2 [✓ (3) $-\frac{1}{2}$] (4) $\frac{1}{2}$

$\frac{-4-1}{4--6} = \frac{-5}{10} = -\frac{1}{2}$

13. What is the solution of the system of equations $c + 3d = 8$ and $c = 4d - 6$?

~~(1)~~ $c = -14$, $d = -2$ (2) $c = -2$, $d = 2$ [✓ (3) $c = 2$, $d = 2$]   ~~(4)~~ $c = 14$, $d = -2$

$c+3d=8$

$c=4d-6$

$4d-6+3d=8$

$7d-6=8$

$7d=14$

$d=2$

$c=4(2)-6$

$c=2$

14.

15. Which equation represents a line that is parallel to the line $y = -4x - 5$?

[✓ (1) $y = -4x + 3$]   (2) $y = -\frac{1}{4}x + 5$ (3) $y = \frac{1}{4}x - 3$ (4) $y = 4x + 5$

16. What is the solution set of the following system of equations?

$x + y = 7$

$x - y = 3$

(1) $\{3,4\}$ [✓ (2) $\{5,2\}$]   (3) $\{10,-3\}$ (4) $\{8,-1\}$

$2x=10$

$x=5$

17. In a linear equation, the independent variable increases at a constant rate, while the dependent variable decreases at a constant rate. The slope of this line is:

(1) zero   [✓ (2) negative]   (3) positive   (4) undefined

18. Which ordered pair is in the solution set of the system of equations $y = -x - 1$ and $y = x^2 + 5x + 6$?

(1) $\{-5,-1\}$ [✓ (2) $\{-5,6\}$]   (3) $(5,-4)$ (4) $\{5,2\}$

use calc

19. Samuel's Car service will charge a flat travel fee of $4.75 for anyone making a trip. They charge an additional set rate of $1.50 per mile that is traveled. Which is an equation that represents the charges?

(1) $y = 1.5x + 1.5$ (2) $y = 4.75x + 4.75$ [✓ (3) $y = 1.5x + 4.75$]   (4) $y = 4.75x + 1.5$

$4.75 + 1.50x = y$

20. Jerome collects stamps. He saved $100 to buy stamps to add to his collection. The stamps cost $1.50, $2, or $5. Which equation models the different ways that Jerome can spend his money where x represents the number of 1.50 stamps, y represents the number of $2 stamps, and z represents the number of $5 stamps?

(1) $7.50x = 100$ (2) $15xyz = 100$ [✓ (3) $1.5x + 2y + 5z = 100$]   (4) $\frac{x}{1.5} + \frac{y}{2} + \frac{z}{5} = 100$

21. What is the solution of the system of equations $2x - 5y = 11$ and $-2x + 3y = -9$?

(1) $\{-3,-1\}$ (2) $\{-1,3\}$ [✓ (3) $\{3,-1\}$]   (4) $\{3,1\}$

$2x-5y=11$

$\underline{-2x+3y=-9}$

$-2y=2$

$y=-1$

$2x-5(-1)=11$

$2x+5=11$

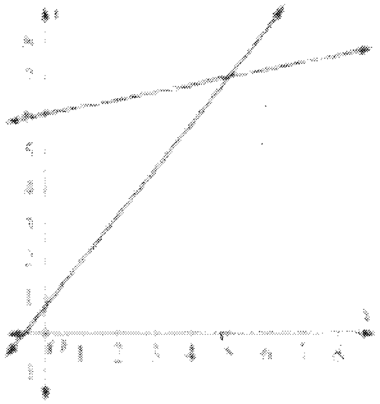
$2x=6$

$x=3$

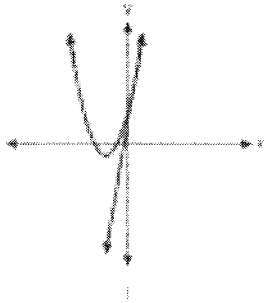
22 The graph below represents a system of linear equations.

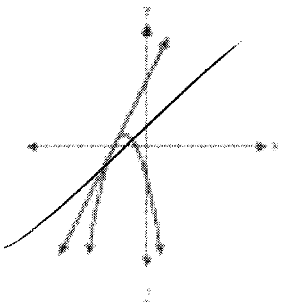
What is the solution set of this system?

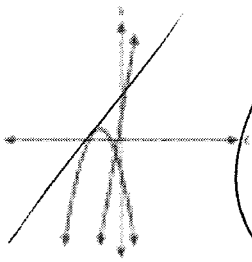
(1) $\left\{\left(0,\frac{3}{4}\right)\right\}$ (2) $\{(0,6)\}$ [✓ (3) $\{(5,7)\}$] (4) $\{(7,5)\}$

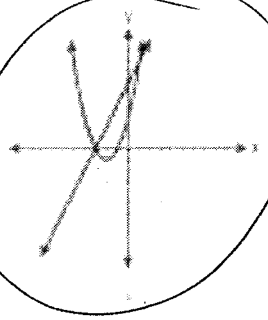


23 Which graph could be used to find the solution to the system $y = 2x + 6$ and $y = x^2 - 4x + 3$?









24 The cost of three notebooks and four pencils is \$8.50. The cost of five notebooks and eight pencils is \$14.50. Determine the cost of one notebook and the cost of one pencil.

Let $n = \#$ notebooks

$p = \#$ pencils

$-2\left(3n + 4p = 8.50\right)$

$5n + 8p = 14.50$

$-6n - \cancel{8p} = -17$

$5n + \cancel{8p} = 14.50$

$-1n = -2.00$

$n = 2.50$

$3(2.50) + 4p = 8.50$

$7.50 + 4p = 8.50$

$4p = 1$

$p = .25$

$\boxed{\$2.50 \text{ notebook} \quad \$.25 \text{ pencil}}$

25 Costco charges $15.00 for membership. Their prices are less than those found in a supermarket. For a gallon of milk, they charge $1.50. The local supermarket charges $3.00 per gallon.

a. Create an equation for the cost of buying x gallons of milk from each of the two stores:

- Cost, C_1, of buying milk from Price Club $C_1 = 1.50x$
- Cost, C_2, of buying milk from supermarket $C_2 = 3.00x$

b. How many gallons of milk would you have to buy in order to have spent the same amount of money at each store?

$15 + 1.50x = 3.00x$

- Gallons: 10 gallons

$x = 10$

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26 Tom throws a ball into the air. The ball travels on a parabolic path represented by the equation, $h = -8t^2 + 32t + 3$, where h is the height, in feet, of the ball, and t is the time in seconds.

a. On the graph below, graph the function from $t = 0$ to $t = 4$ seconds.

b. What is the value of t at which h has its greatest value? 35 sec

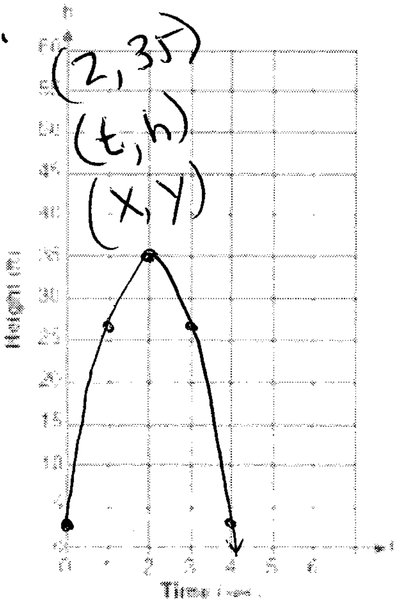
how long until it hits the ground?.

$h = -8t^2 + 32t + 3$

$a = -8$
$b = 32$
$c = 3$

$$\frac{-32 \pm \sqrt{(32)^2 - 4(-8)(3)}}{2(-8)}$$

$$x = \overset{-.09}{\boxed{4.09}}$$



0	3
1	27
2	35
3	27
4	3

$$x = \frac{-b}{2a}$$

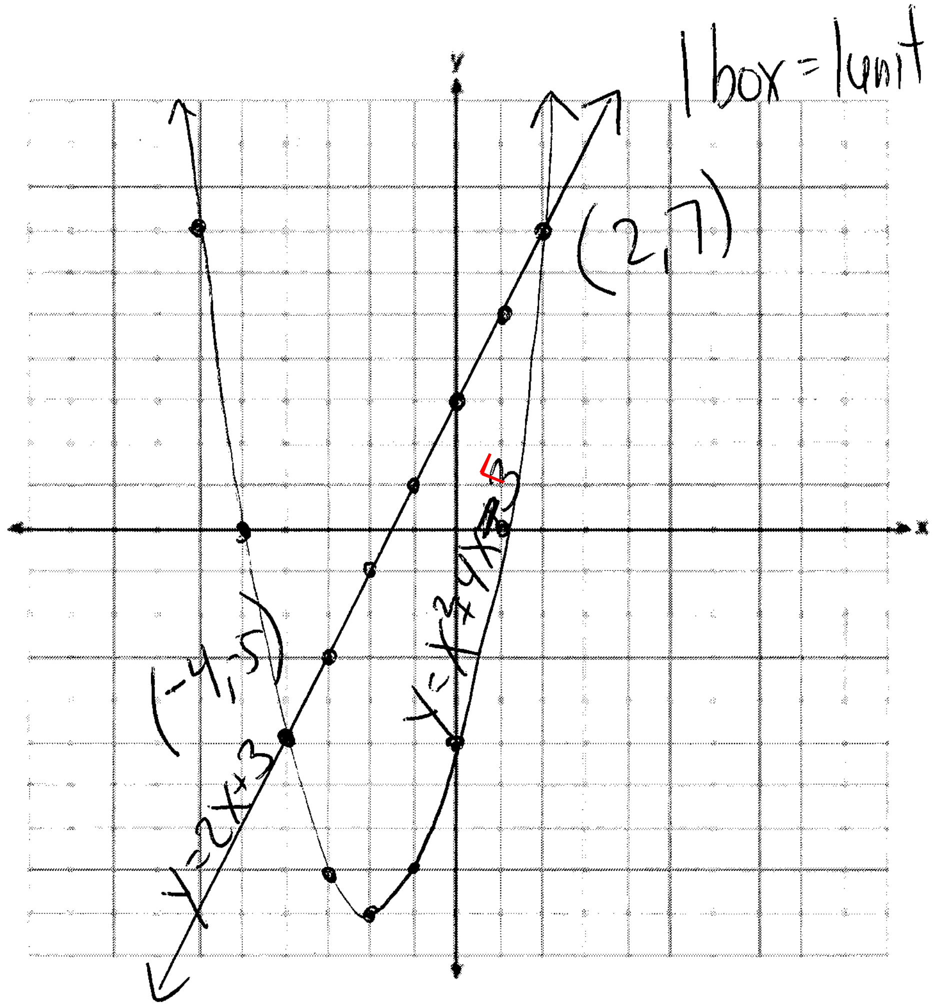
$$x = \frac{-32}{2(-8)} = 2$$

$$y = -8t^2 + 32t + 3$$
$$= -8(2)^2 + 32(2) + 3$$
$$= 35 \text{sec}$$

27. On the set of axes below, solve the following system of equations graphically for all values of x and y. State the coordinates of all the solutions.

$$y = x^2 + 4x - 5$$
$$y = 2x + 3$$

X	Y
-5	0
-4	-5
-3	-8
-2	-9
-1	-8
0	-5
1	0



REVIEW OF FACTORING

STEPS IN FACTORING:

Step 1: Factor out the greatest common factor (GCF). (There will not always be one).

Ex: $9x^2 - 3x = 3x(3x - 1)$

Step 2: Count the number of terms.

if **Two terms**: Look to see if you have a Difference of Two Squares.

Difference of Two Squares:

$x^2 - y^2 = (x + y)(x - y)$ $\qquad$ $x^2 + y^2 \rightarrow$ PRIME

$4x^2 - 9y^4 = (2x + 3y^2)(2x - 3y^2)$

if **Three terms**: Look for two binomials.

If a = 1; you are looking for two numbers that multiply together to get "c" and also add to get "b"

Ex: $x^2 + 6x - 16$ $\qquad$ Prod = -16 $\qquad$ Sum = 6

$\boxed{(x + 8)(x - 2)}$ $\qquad$ 8 ^ -2 $\qquad$ 8 ^ -2

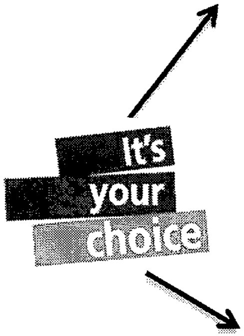
If a ≠1; Multiply "a" and "c" , this will be the new product. Now you are looking for two numbers that multiply to get "ac" and add to get "b"

Ex: $2x^2 - x - 10$ $\qquad$ Prod = -20 $\qquad$ Sum = -1

$\frac{(2x \cancel{+ 4})(2x - 5)}{\cancel{2}}$ $\qquad$ 4 ^ -5 $\qquad$ 4 ^ -5

$\boxed{(x + 4)(2x - 5)}$

$ax^2 + bx + c$



Clues for factoring trinomials by **trial and error:**

- If the sign of the last term is +, the middle sign of the binomials will have the same sign as the second term in the trinomial.
  Example: $x^2 - 3x + 2 = (x - 2)(x - 1)$ or $x^2 + 5x + 6 = (x + 3)(x + 2)$
- If the sign of the last term is -, the middle sign of the binomials will be + and -.
  Example: $x^2 - 5x - 6 = (x - 6)(x + 1)$ or $x^2 + x - 56 = (x + 8)(x - 7)$

Check factors/answer on the home screen in equation form.

Factor x^2 - 12x + 36 OR Find (x - 6)^2

- Choose your "favorite" positive one-digit (for ease) integer value and store the value in *x* (do not pick 0 or 1). For example, to store a 7: **7 STO▶ x**
- Hit **ENTER.**
- Enter the problem to be factored (or multiplied) and set "=" to one of the possible answers (or the answer you want to check). The "=" sign is under **2nd MATH (TEST) #1 =.**
- Hit **ENTER.**
- If a **0** appears, this is NOT the correct answer.
  If a 1 appears, this IS the correct answer.

1.

Written in simplest factored form, the binomial $2x^2 - 50$ can be expressed as

(1) $2(x-5)(x-5)$ (3) $(x-5)(x+5)$
[✓ (2) $2(x-5)(x+5)$] (4) $2x(x-50)$

$2(x^2-25)$
$2(x+5)(x-5)$

Show how to check on calc

2.

What are the factors of $x^2 - 5x + 6$?

(1) $(x+2)$ and $(x+3)$ (3) $(x+6)$ and $(x-1)$
[✓ (2) $(x-2)$ and $(x-3)$] (4) $(x-6)$ and $(x+1)$

$P = 6$ -3 -2
$S = -5$

3.

Factored completely, the expression $2x^2 + 10x - 12$ is equivalent to

(1) $2(x-6)(x+1)$ (3) $2(x+2)(x+3)$
[✓ (2) $2(x+6)(x-1)$] (4) $2(x-2)(x-3)$

$2(x^2+5x-6)$
$2(x+6)(x-1)$

4.

Factored completely, the expression $2y^2 + 12y - 54$ is equivalent to

[✓ (1) $2(y+9)(y-3)$] (3) $(y+6)(2y-9)$
(2) $2(y-3)(y-9)$ (4) $(2y+6)(y-9)$

$2(y^2+6y-27)$
$2(y+9)(y-3)$

5.

Expressed in factored form, the binomial $4a^2 - 9b^2$ is equivalent to

(1) $(2a-3b)(2a-3b)$ (3) $(4a-3b)(a+3b)$
[✓ (2) $(2a+3b)(2a-3b)$] (4) $(2a-9b)(2a+b)$

$(2a+3b)(2a-3b)$

6.

Factor completely: $3x^2 - 27$

(1) $3(x-3)^2$ [✓ (3) $3(x+3)(x-3)$]
(2) $3(x^2-27)$ (4) $(3x+3)(x-9)$

$3(x^2-9)$
$3(x+9)(x-9)$

7.

Which expression is a factor of $n^2 + 3n - 54$?

(1) $n+6$ (3) $n-9$
(2) n^2+9 [✓ (4) $n+9$]

$(n+9)(n-6)$

8.

Factored, the expression $16x^2 - 25y^2$ is equivalent to

[✓ (1) $(4x-5y)(4x+5y)$] (3) $(8x-5y)(8x+5y)$
(2) $(4x-5y)(4x-5y)$ (4) $(8x-5y)(8x-5y)$

$(4x-5y)(4x+5y)$

9.

The expression $x^2 - 16$ is equivalent to

(1) $(x+2)(x-8)$ [✓ (3)] $(x+4)(x-4)$
(2) $(x-2)(x+8)$ (4) $(x+8)(x-8)$

10.

The expression $9x^2 - 100$ is equivalent to

(1) $(9x-10)(x+10)$ (3) $(3x-100)(3x-1)$
[✓ (2)] $(3x-10)(3x+10)$ (4) $(9x-100)(x+1)$

$(3x-10)(3x+10)$

11.

~~The expression $x^2 - 16$ is equivalent to~~

~~(1) $(x+2)(x-8)$ (3) $(x+4)(x-4)$~~
~~(2) $(x-2)(x+8)$ (4) $(x+8)(x-8)$~~

12.

The greatest common factor of $4a^2b$ and $6ab^3$ is

[A] $24a^3b^4$ [B] $2ab^2$
[✓ [C] $2ab$] [D] $12ab$

$2ab$

13.

What are the factors of $x^2 - 10x - 24$?

[✓ [A]] $(x-12)(x+2)$ [B] $(x+12)(x-2)$
[C] $(x-4)(x+6)$ [D] $(x-4)(x-6)$

$(x-12)(x+2)$

14.

If one factor of $56x^4y^3 - 42x^2y^6$ is $14x^2y^3$, what is the other factor?

[✓ [A] $4x^2 - 3y^3$]　[B] $4x^2 - 3y^2$

[C] $4x^2y - 3xy^2$　[D] $4x^2y - 3xy^3$

$14x^2y^3(4x^3 - 3y^3)$

15.

When factored completely, $x^3 + 3x^2 - 4x - 12$ equals

1) $(x+2)(x-2)(x-3)$
[✓ 2) $(x+2)(x-2)(x+3)$]
3) $(x^2-4)(x+3)$
4) $(x^2-4)(x-3)$

Factor by groping

$x^2(x+3) - 4(x+3)$

$(x^2-4)(x+3)$

$(x+2)(x-2)(x+3)$

Careful

16.

When factored completely, the expression $3x^3 - 5x^2 - 48x + 80$ is equivalent to

1) $(x^2-16)(3x-5)$
2) $(x^2+16)(3x-5)(3x+5)$
[✓ 3) $(x+4)(x-4)(3x-5)$]
4) $(x+4)(x-4)(3x-5)(3x-5)$

$x^2(3x-5) - 16(3x-5)$

$(x^2-16)(3x-5)$

$(x+4)(x-4)(3x-5)$

17.

The expression $x^2(x+2) - (x+2)$ is equivalent to

1) x^2
2) $x^2 - 01$
3) $x^3 + 2x^2 - x + 2$
[✓ 4) $(x+1)(x-1)(x+2)$]

$(x^2-1)(x+2)$

$(x+1)(x-1)(x+2)$

18.

$12x^2 + 2 + 11x$

[A] $(3x-2)(4x-1)$　[B] $(3x-2)(4x+1)$

[C] $(3x+2)(4x-1)$　[✓ [D] $(3x+2)(4x+1)$]

$12x^2 + 11x + 2$

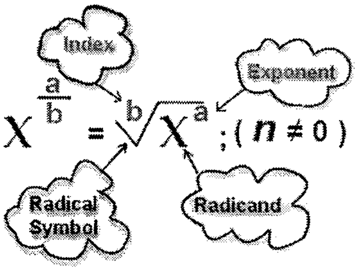
$p = 24 <^{8}_{3}$

$s = 11$

$(3x+2)$　$(4x+1)$

$$\frac{(\cancel{12x+8})(12x+3)}{\cancel{12}\ \cancel{4}\cdot 3}$$

REVIEW OF RADICALS



Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169…

Simplifying Radicals: find two numbers that multiply to the number under the radical where one number ***must*** be a perfect square (look for the largest perfect square).

Simplify:

$$\sqrt{48}=\sqrt{16\cdot 3}=\sqrt{16}\cdot\sqrt{3}=4\sqrt{3}$$

$\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$

The square root of a product is equal to the product of the square roots of each factor.

Adding/Subtracting: Two radicals must have the SAME radicand. If so, add/subtract coefficients and leave the common radical alone.

Ex: **Add:** $2\sqrt{3}+5\sqrt{3}$

Answer: $7\sqrt{3}$

When adding or subtracting radicals, you must use the same concept as that of adding or subtracting "like" variables.

Multiplying/Dividing: Any two radicals can multiply/divide (do not have to be the same radicand). Multiply/Divide the coefficients and multiply/divide the radicands.

Ex: $2\sqrt{3} * 4\sqrt{5} = 2 * 4\sqrt{3*5} = 8\sqrt{15}$

Ex: $\frac{4\sqrt{15}}{2\sqrt{3}}=\frac{4}{2}\cdot\sqrt{\frac{15}{3}}=2\sqrt{5}$

Ex: $\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{\sqrt{9}}=\frac{2\sqrt{3}}{3}$

To "remove" a radical from the denominator, multiply the top and bottom of the fraction by that same radical to create a rational number (a perfect square radical) in the denominator.
*This process is called **rationalizing the denominator.***

An expression under a radical sign is in **simplest radical** form when:

1) there is no integer under the radical sign with a perfect square factor,
2) there are no fractions under the radical sign,
3) there are no radicals in the denominator

Regents Review

1\.

The expression $\sqrt{50}$ can be simplified to

[✓ (1) $5\sqrt{2}$]
(3) $2\sqrt{25}$
(2) $5\sqrt{10}$
(4) $25\sqrt{2}$

$\sqrt{25}\sqrt{2}$

$5\sqrt{2}$

2\.

What is $\frac{\sqrt{32}}{4}$ expressed in simplest radical form?

[✓ (1) $\sqrt{2}$]
(3) $\sqrt{8}$
(2) $4\sqrt{2}$
(4) $\frac{\sqrt{8}}{2}$

$\frac{\sqrt{16}\sqrt{2}}{4}$ $\frac{4\sqrt{2}}{4}$ $\sqrt{2}$

3\.

The expression $\sqrt{93}$ is a number between

(1) 3 and 9
[✓ (3) 9 and 10]
(2) 8 and 9
(4) 46 and 47

$\sqrt{81}$ $\sqrt{100}$

9 10

4\.

What is 3 √250 expressed in simplest radical form?

1) 5 √10
2) 8√ 10
[✓ 3) 15 √10]
4) 75 √10

$3\sqrt{25}\sqrt{10}$

$3 \cdot 5\sqrt{10}$

$15\sqrt{10}$

5\.

When $5\sqrt{20}$ is written in simplest radical form, the result is $k\sqrt{5}$. What is the value of k?

1) 20
[✓ 2) 10]
3) 7
4) 4

$5\sqrt{20}$

$5\sqrt{4}\sqrt{5}$

$5 \cdot 2\sqrt{5}$

$10\sqrt{5}$

6\.

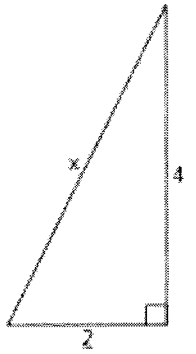
Express $4\sqrt{75}$ in simplest radical form.

$4\sqrt{25}\sqrt{3}$

$4 \cdot 5\sqrt{3}$

$\boxed{20\sqrt{3}}$

7\. Theo determined that the correct length of the hypotenuse of the right triangle in the accompanying diagram is $\sqrt{20}$. Fiona found length of the hypotenuse to be $2\sqrt{5}$. Is Fiona answer also correct? Justify your answer.



$\sqrt{20}$

$\sqrt{4}\sqrt{5}$

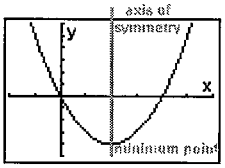
$2\sqrt{5}$

Yes both are equal

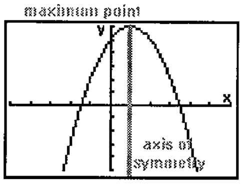
REVIEW OF QUADRATICS

Equation of a Parabola (Standard Form): $f(x) = ax^2 + bx + c$

(Vertex Form): $f(x) = a(x-h)^2 + k$, **vertex is (h,k)**



If *a* is positive, the parabola opens upward and has a minimum point.



If *a* is negative, the parabola opens downward and has a maximum point.

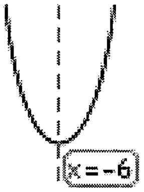
Axis of Symmetry: the vertical line that passes through the vertex

Find the Axis of Symmetry:

$y=x^2+12x+32$

$a=1 \quad b=12 \quad c=32$

$x=\frac{-b}{2a}=\frac{-12}{2(1)}=-6$



$$x = \frac{-b}{2a}$$

Vertex (turning point): find the x-value by using the axis of symmetry formula then plug that x-value into the parabola's equation to find the y-value.

Roots (x-intercepts, zeros of the function): The values of x where the graph intersects the x-axis (y-value = 0). A parabola can have 2, 1, or no roots.

To find the roots algebraically:

Step 1: Set the equation equal to zero

Step 2: FACTOR (GCF, DOTS, or Trinomial),
Complete the Square, or
Use the quadratic formula

Don't forget to check with the TI-84 (Enter equation into y1)

Solve by Factoring:

If "bx" is missing…

$x^2-81=0$

$(x-9)(x+9)=0$

$x-9=0$ or $x+9=0$

$x=9 \quad x=-9$

$\{-9, 9\}$

OR

$x^2-81=0$

$x^2=81$

$\sqrt{x^2}=\pm\sqrt{81}$

$x=\pm 9$

$\{-9, 9\}$

Solve by Factoring

$x^2+12x+32=0$

$(x+4)(x+8)=0$

$x+4=0 \quad x+8=0$

$x=-4 \quad x=-8$

$\cancel{1 \quad 32} = 33$

$\cancel{2 \quad 16} = 18$

$\boxed{4 \quad 8} = 12$

$(-4)^2+12(-4)+32$

$=16-48+32=0$ ✓

$(-8)^2+12(-8)+32$

$=64-96+32=0$ ✓

Solve by Completing the Square:

1. Be sure that the coefficient of the highest power is one. If it is not, divide each term by that value to create a leading coefficient of one.	$x^2+8x-4=0$
2. Move the constant term to the right hand side.	$x^2+8x=4$
3. Prepare to add the needed value to create the perfect square trinomial. Be sure to balance the equation. The boxes may help you remember to balance.	$x^2+8x+\square=4+\square$
4. To find the needed value for the perfect square trinomial, take half of the coefficient of the *middle term* (*x*-term), square it, and add that value to both sides of the equation. Take half and square $\downarrow$ $x^2+8x+\square=4+\square$	$x^2+8x+\boxed{16}=4+\boxed{16}$
5. Factor the perfect square trinomial.	$(x+4)^2=20$
6. Take the square root of each side and solve. Remember to consider both plus and minus results.	$x+4=\pm\sqrt{20}$ $x=-4\pm\sqrt{20}=-4\pm2\sqrt{5}$ $x=-4+2\sqrt{5}$ $x=-4-2\sqrt{5}$

Solve by using the quadratic formula:

$$y = x^2 + 2x - 3$$

$$\frac{-2\pm\sqrt{(2)^2-4(1)(-3)}}{2(1)}$$

$$\downarrow$$

$$\frac{-2\pm\sqrt{4+12}}{2}\rightarrow\frac{-2\pm\sqrt{16}}{2}$$

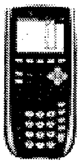
$$\frac{-2\pm4}{2}\begin{matrix}\nearrow & \frac{-2+4}{2}\rightarrow\frac{2}{2}\rightarrow1\\ \searrow & \frac{-2-4}{2}\rightarrow\frac{-6}{2}\rightarrow-3\end{matrix}$$

The Quadratic Formula ...

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For Quadratic Equations
$ax^2+bx+c=0$

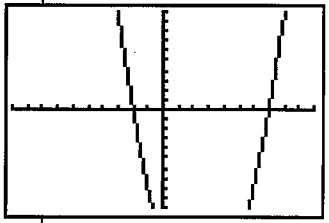
Discriminant: b^2-4ac
if positive – 2 solutions
If negative – No sol. (imaginary)
If zero -- 1 solution



Solve: $x^2 - 5x - 14 = 0$

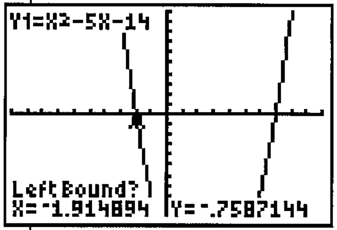
Since this equation is set equal to zero, the roots will be the locations where the graph crosses the *x*-axis (if the roots are real numbers).
(Remember that the *x*-axis is really just $y = 0$.)

1. Set $Y_1 = x^2 - 5x - 14$



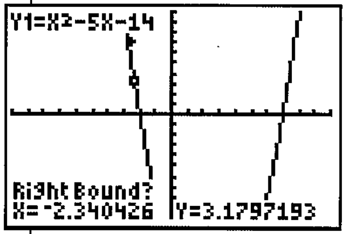
2. Use the ZERO command to find the roots -- 2nd TRACE (CALC), #2 zero

3. Left bound? Move the spider as close to the root (where the graph crosses the *x*-axis) as possible. Hit the left arrow to move to the "left" of the root. Hit ENTER. A "marker" ▶ will be set to the left of the root.



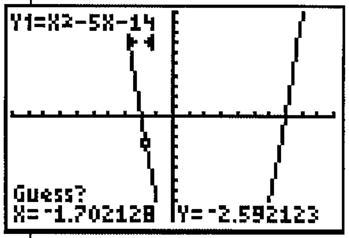
```
Y1=X²-5X-14
Left Bound?
X=-1.914894   Y=-.7587144
```

4. Right bound? Move the spider as close to the root (where the graph crosses the *x*-axis) as possible. Hit the right arrow to move to the "right" of the root. Hit ENTER. A "marker" ◀ will be set to the right of the root.



```
Y1=X²-5X-14
RightBound?
X=-2.340426   Y=3.1797193
```

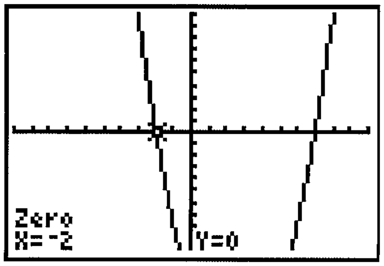
5. Guess? Just hit ENTER.



```
Y1=X²-5X-14
Guess?
X=-1.702128   Y=-2.592123
```

6. Repeat the entire process to find the second root (which in this case happens to be $x = 7$).

Answer: one of the roots is $x = -2$

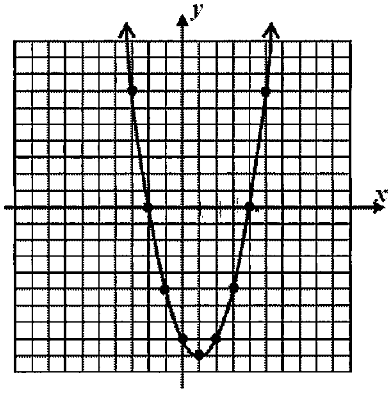


```
Zero
X=-2   Y=0
```

Regents Review

1) Each graph shown below represents a quadratic function of the form $y = x^2 + bx + c$. Use the graph to determine the zeros of the function. Then determine the binomial factors of the function and express the quadratic function in its $y = x^2 + bx + c$ form.

(a)



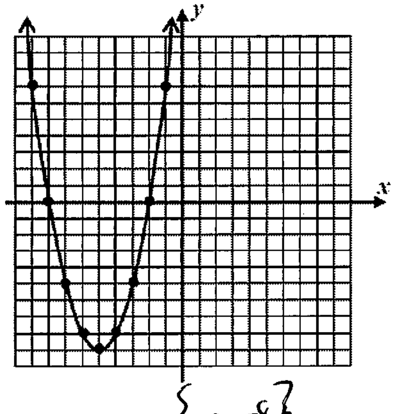
Zeros: $\{2, 4\}$

Factors: $(x+2)(x-4)$

Equation: $y = x^2 - 4x + 2x - 8$

$y = x^2 - 2x - 8$

(b)



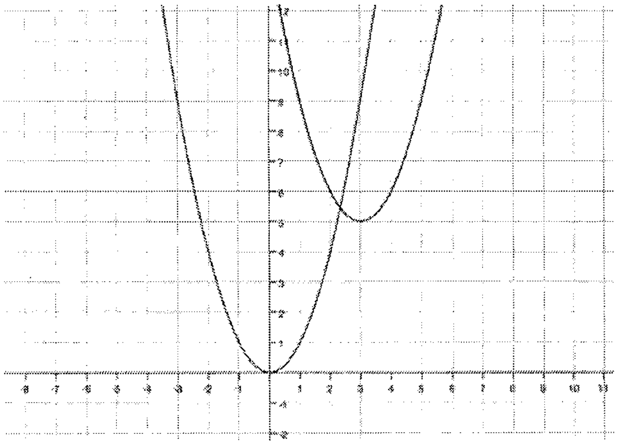
Zeros: $\{-2, -8\}$

Factors: $(x+8)(x+2)$

Equation: $y = x^2 + 10x + 16$

---

2) Study the graph below. Identify the parent function. Then write a description and equation for the transformed function.



vertex formula

Parent Function $y = x^2$

Transformed Equation

Shifted 3 Right 5 up (3,5)

Description of transformaion:

$\underline{y = a(x-h)^2 + k}$

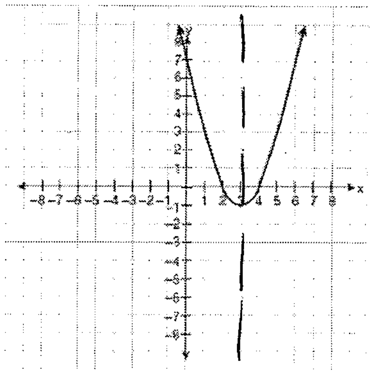
$y = (x-3)^2 + 5$

$y = (x-3)(x-3) + 5$

$y = x^2 - 6x + 9 + 5$

$\underline{y = x^2 - 6x + 14}$

3) Which is an equation of the line of symmetry for the parabola in the accompanying diagram?
(1) $x=2$ (2) $x=4$ [✓ (3) $x=3$] (4) $y=3$



4) Which of the following sets of ordered pairs does not represent a function?
(1) $\{(1,2),(3,4),(4,5),(6,7)\}$
[✓ (2) $\{(0,3),(1,5),(3,9),(1,13)\}$]
(3) $\{(-2,2),(-3,3),(-4,-4),(5,-5)\}$
(4) $\{(1,2),(2,2),(3,5),(4,8)\}$

5) How many roots does $x^2+7x+6=0$ have?
(1) 1 [✓ (2) 2] (3) 3 (4) 0

b^2-4ac
$7^2-4(1)(6)$
25
2 roots

6) Consider the graph of the function $f(x)=ax^2+bx+c$. If the coefficient of x^2 were 3, what would be true of the new function?
~~(1)~~ The vertex would be 3 units above the vertex of the original parabola.
(2) The new parabola would be 3 units to the right of the original parabola.
(3) The new parabola would be wider than the original parabola.
[✓ (4) The new parabola would be narrower than the original parabola.]

7) What is minimum point of the graph of the equation $y=2x^2+8x+9$?
(1) $(2,33)$ (2) $(2,17)$ (3) $(-2,-15)$ [✓ (4) $(-2,1)$]

vertex

$2x^2+8x=-9$
$x^2+4x=-4.5$
$x^2+4x+4=-4.5+4$
$(x+2)^2=-.5$
$y=2(x+2)^2+(.5)$
$y=2(x+2)^2+1$
$(-2,1)$

8) If the roots of a quadratic equation are -2 and 3, its equation can be written as
(1) $(x-2)(x+3)=0$ [✓ (2) $(x+2)(x-3)=0$] (3) $(x+2)(x+3)=0$ (4) $(x-2)(x-3)=0$

9) Which describes the translation from $y=x^2$ to $y=(x+\overset{h}{2})^2\overset{k}{-1}$?
(1) Up 2 units and right 1 unit.
(2) Down 1 unit and right 2 units.
[✓ (3) Down 1 unit and left 2 units.]
(4) Up 1 unit and left 2 units

$y=a(x-h)^2+k$

k ↑+ ↓-
h → neg ← pos

h k
+2 -1
left down

Y intercept
↓

10) A ball is thrown in the air. Its height, h in meters, is given by $h = -4.9t^2 + 30t + 6$, where t is the time in seconds. What is the height of the ball at the instant it is thrown?
(1) 0 m (2) 4.9 m [✓ (3) 6 m] (4) 30 m

11) The table contains values for x and y in a quadratic function.
What are the roots of the function?
[✓ (1) −1 and 5] (2) −1 and 10 ~~(3) −1, 0, and 5~~ ~~(4) −1, 10, and 5~~

x	y
−1	0
0	10
1	16
2	18
3	16
4	10
5	0

12) Which ordered pair cannot be a solution of $h(t) = -16t^2 + 80t$, if h is the height of a ball above the ground after t seconds?
(1) $(1,64)$ (2) $(2,96)$ [✓ (3) $(-4,256)$] (4) $(5,0)$

13) A rocket is launched from the ground. The function $h(t) = -4.9t^2 + 180t$ models the height of the rocket. If all other factors remain the same, which of the following functions models the height of a rocket above the ground if it is launched from a platform 100 feet in the air?
(1) $h(t) = -4.9t^2 + 280t$
[✓ (3) $h(t) = -4.9t^2 + 180t + 100$]
(2) $h(t) = -4.9t^2 + 180t - 100$
(4) $h(t) = -4.9t^2 + 180(t + 100)$

14) A bottle rocket that was made in science class had a trajectory path that followed the quadratic equation, $f(x) = -x^2 + 4x + 6$. What is the turning point of the rocket's path?
(1) $(1,5)$ [✓ (2) $(2,10)$] (3) $(-2,-10)$ (4) $(1,-5)$

15. What is the average rate of the function $f(x) = x^2 + 6x + 9$ on the interval $-1 \le x \le 3$?
(1) −4 (2) −8 [✓ (3) 8] (4) 4

X	Y	
−1	4	5
0	9	7
1	16	9
2	25	11
3	36	

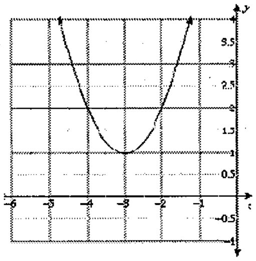
16. Which statement *is not* supported by the graph shown?

(1) The vertex is $(-3,1)$. ✓

[✓ (2) The roots are −4 and −2.]

(3) The coefficient of x^2 in the equation is positive.

(4) The quadratic function has no real roots.



$y=-1(x+3)^2+1$

$y=-(x+3)^2+1$

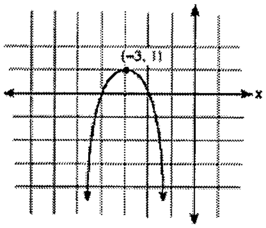
17) Which equation represents the parabola shown in the accompanying graph?

(1) $f(x)=(x+1)^2-3$

[✓ (2) $f(x)=-(x+3)^2+1$]

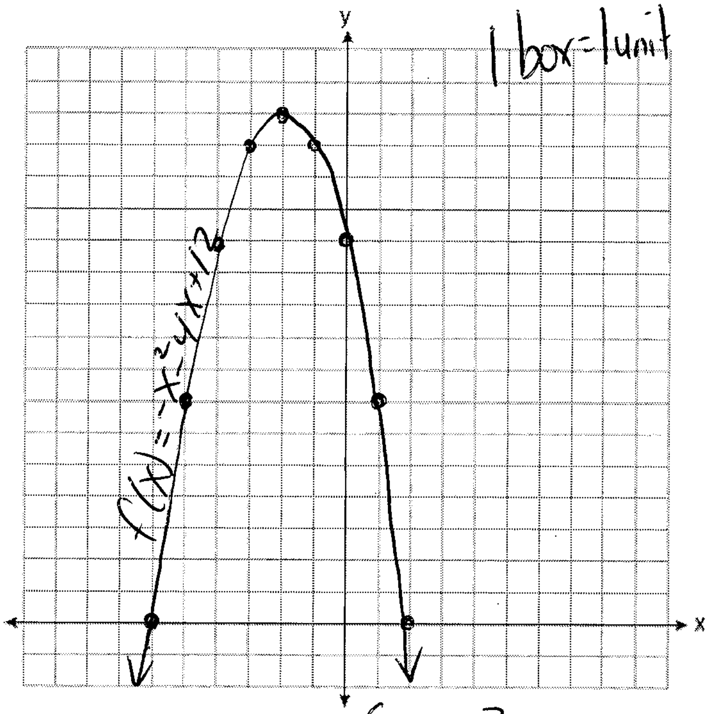
(3) $f(x)=-(x-3)^2+1$

(4) $f(x)=-(x-3)^2-3$



18) On the set of axes below graph the following quadratic function, $f(x)=-x^2-4x+12$.

Use as the domain $\{x|-6\le x\le 2\}$. Then answer the questions that follow. NO ARROWS



TABLE

x	$f(x)$
-6	0
-5	7
-4	12
-3	15
-2	16
-1	15
0	12
1	7
2	0

a. What are the x-intercepts? $\{-6, 2\}$

b. What is the axis of symmetry? $x=-2$

c. What is the vertex of the graph? $(-2, 16)$

d. Is the this a maximum or minimum value max

19) On the set of axes below. solve the following system of equations graphically for all values of x and y. Notice the quadratic is shown in vertex form.

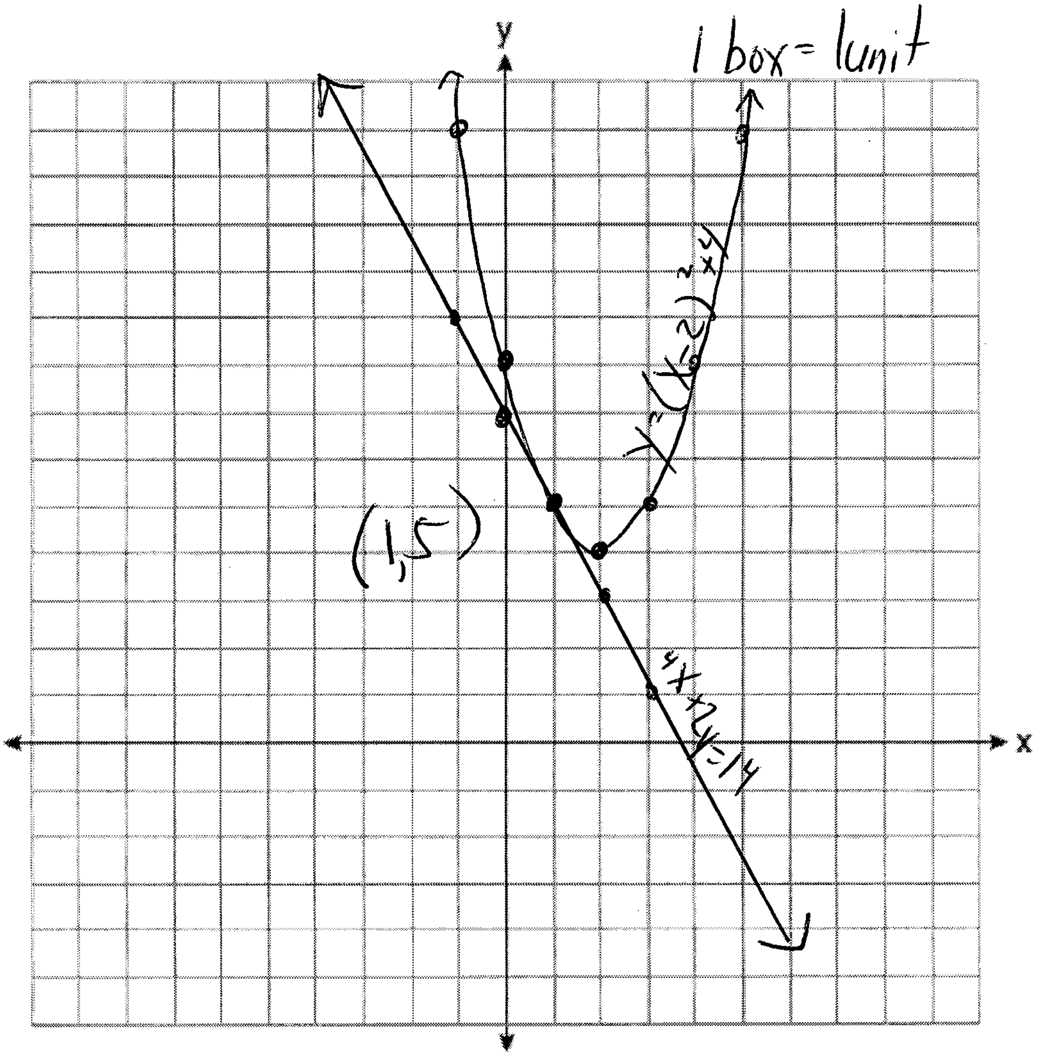
$$y = (x-2)^2 + 4$$

$$4x + 2y = 14$$

$y = (x-2)^2 + 4$

X	Y
-1	13
0	8
1	5
2	4
3	5
4	8
5	13

1 box = 1unit



$4\cancel{x}+2y=14$

$-\cancel{4}x \quad -4x$

$\frac{2y}{2} = \frac{-4x}{2} + \frac{14}{2}$

$y = -2x + 7$

SOLVE FOR X:

1. $x^2+10x+16 = 0$

$(x+8)(x+2)=0$

$x=-8$ $x=-2$

$\{-2,-8\}$

2. $x^2-11x+24=0$

$(x-8)(x-3)=0$

$x=8$ $x=3$

$\{3,8\}$

3. $x^2+6x-16=0$

$(x+8)(x-2)=0$

$x=-8$ $x=2$

$\{8,-2\}$

4. $x^2+7x-30=0$

$(x+10)(x-3)=0$

$\{-10,3\}$

5. $4x^2-81=0$

$(2x+9)(2x-9)=0$

$\{+\frac{9}{2},-\frac{9}{2}\}$

6. $4x^2-49=0$

$(2x+7)(2x-7)=0$

$x=\pm\frac{7}{2}$

$\{\pm\frac{7}{2}\}$

7. $x^2+5x=0$

$x(x+5)=0$

$x=0 \mid x=-5$

$\{-5,0\}$

8. $2x^2+x=0$

$x(2x+1)=0$

$x=0$ $x=-\frac{1}{2}$

$\{0,-\frac{1}{2}\}$

9. $2x^2+18x+40=0$

$2(x^2+9x+20)=0$

$2(x+4)(x+5)=0$

$2\neq0 \mid x=-4 \mid x=-5$

$\{-4,-5\}$

10. $4x^2-24x+36=0$

$4(x^2-6x+9)=0$

$4(x-3)(x-3)=0$

$4\neq0 \mid x=3 \mid x=3$

$\{3\}$

11. $5x^2 - 35x - 90 = 0$

$5(x^2-7x-18)=0$

$5(x-9)(x+2)=0$

$\{9,-2\}$

12. $3x^2 + 15x - 108 = 0$

$3(x^2+5x-36)=0$

$3(x+9)(x-4)=0$

$\{-9, 4\}$

13. $2x^2 + 7x + 6 = 0$

$P=12 < {4 \atop 3}$

$S=7$

$\frac{(2x+4)(2x+3)}{2}=0$

$(x+2)(2x+3)$

$\left\{-\frac{3}{2},-2\right\}$

$x=-2 \quad x=-\frac{3}{2}$

14. $3x^2 + 8x + 4 = 0$

$P=12 < {6 \atop 2}$

$S=8$

$\frac{(\cancel{3x+6})(3x+2)}{3}=0$

$(x+2)(3x+2)=0$

$\left\{-\frac{2}{3},-2\right\}$

$x=-2 \quad x=-\frac{2}{3}$

15. $2x^2 + 5x + 3 = 0$

$P=6 < {3 \atop 2}$

$S=5$

$\frac{(2x+3)(\cancel{2x+2})}{\cancel{2}}=0$

$(2x+3)(x+1)$

$\left\{-1,-\frac{3}{2}\right\}$

$x=-\frac{3}{2} \quad x=-1$

16. $5x^2 + 16x + 3 = 0$

$P=15$

$S=+16$

$\frac{(\cancel{5x+15})(5x+1)}{\cancel{5}}=0$

$(x+3)(5x+1)=0$

$\left\{-3,-\frac{1}{5}\right\}$

$x=-3 \quad x=-\frac{1}{5}$

17. Solve by completing the square:

$x^2 + 6x - 7 = 0$

$\left(\frac{6}{2}\right)^2$

$x^2+6x+9=7+9$

$\sqrt{(x+3)^2}=\sqrt{16}$

$x+3=\pm 4$

$-3 \quad -3$

$x=4-3 \quad x=-4-3$

$x=1 \quad x=-7$

$\{-7,1\}$

18. Solve by completing the square:

$x^2 + 16x + 15 = 0$

$\left(\frac{16}{2}\right)^2$

$x^2+16x+64=-15+64$

$\sqrt{(x+8)^2}=\sqrt{49}$

$x+8=\pm 7$

$-8 \quad -8$

$x=7-8 \quad x=-7-8$

$-1 \quad -15$

$\{-15,-1\}$

19. Solve by completing the square: $x^2 + 4x - 14 = 0$ $x^2+4x+4 = 14+4$ $\sqrt{(x+2)^2} = \sqrt{18}$ $x+2 = \sqrt{18}$ $x+2 = \pm 3\sqrt{2}$ $-2 \quad -2$ $\boxed{x = -2 \pm 3\sqrt{2}}$ $\sqrt{18}$ $\sqrt{9}\ \sqrt{2}$ $3\sqrt{2}$	20. Solve by completing the square: $x^2 + 10x - 3 = 0$ $\left(\frac{10}{2}\right)^2$ $x^2+10x+25 = 3+25$ $\sqrt{(x+5)^2} = \sqrt{28}$ $x+5 = \pm\sqrt{28}$ $x+5 = \pm 2\sqrt{7}$ $-5 \quad -5$ $\boxed{x = -5 \pm 2\sqrt{7}}$ $\sqrt{28}$ $\sqrt{4}\ \sqrt{7}$

Express each of the following irrational numbers in simplest radical form:

(a) $\sqrt{50}$ $\boxed{5\sqrt{2}}$

(b) $\sqrt{72}$ $\sqrt{36}\ \sqrt{2}$ $\boxed{6\sqrt{2}}$

(c) $\sqrt{54}$ $\sqrt{9}\ \sqrt{6}$ $\boxed{3\sqrt{6}}$

Solve the following equations using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

21. $x^2 + 8x - 4 = 0$

$a = 1$

$b = 8$

$c = -4$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)}$$

$\sqrt{80}$

$\sqrt{16}\ \sqrt{5}$

$4\sqrt{5}$

$$x = \frac{-8 \pm \sqrt{80}}{2}$$

$$x = \frac{-8+4\sqrt{5}}{2} \qquad x = \frac{-8-4\sqrt{5}}{2}$$

$$x = -4+2\sqrt{5} \qquad x = -4-2\sqrt{5}$$

$.472 \qquad -8.472$

22. $x^2 - 6x - 1 = 0$

$a = 1$

$b = -6$

$c = -1$

$$x = \frac{+6 \pm \sqrt{(-6)^2 - 4(1)(-1)}}{2a}$$

$$x = \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm 2\sqrt{10}}{2}$$

$$x = \frac{6+2\sqrt{10}}{2} \qquad x = \frac{6-2\sqrt{10}}{2}$$

$= 3+\sqrt{10} \qquad = 3-\sqrt{10}$

$= 6.16 \qquad = -.16$

23. $3x^2 - 10x + 5 = 0$

$a = 3$

$b = -10$

$c = 5$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{100 - 60}}{2(3)}$$

$$= \frac{10 \pm \sqrt{40}}{6}$$

$\sqrt{40}$

$\sqrt{4}\ \sqrt{10}$

$2\sqrt{10}$

$$= \frac{\overset{5}{\cancel{10}} \pm \overset{1}{\cancel{2}}\sqrt{10}}{\overset{3}{\cancel{6}}}$$

$$= \frac{5 \pm \sqrt{10}}{3}$$

$$= \frac{5 + \sqrt{10}}{3} \qquad \frac{5 - \sqrt{10}}{3}$$

24. Place in standard form and solve.

$x^2 + 5x - 5 = 3x + 10$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5 \quad x = 3$

25. Place in standard form and solve

$x^2 + 7x + 24 = -2x + 4$

$x^2 + 9x + 20 = 0$

$(x + 5)(x + 4) = 0$

$x = -5 \quad x = -4$

$\{-5, -4\}$

26. $\frac{x+2}{2} = \frac{12}{x}$

$x(x + 2) = 24$

$x^2 + 2x - 24 = 0$

$(x + 6)(x - 4) = 0$

$\{-6, 4\}$

27. $\frac{8}{x} = \frac{x+2}{3}$

$x(x + 2) = 24$

$x^2 + 2x = 24$

$\{-6, 4\}$

28. The square of a number increased by 3 times the number equals 4. Find the number.

let X= a number

$x^2+3x=4$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$\{-4,1\}$

29. Find two pairs of consecutive odd integers whose product is 63.

let x - 1st

x+2 - 2nd

$x(x+2)=63$

$x^2+2x=63$

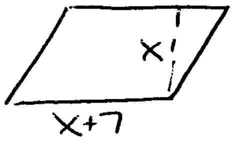
$x^2+2x-63=0$

$(x+9)(x-7)=0$

$\{-9,7\}$

$\boxed{\begin{matrix} x=-9 & 7 \\ x+2=-7 & 9 \end{matrix}}$

30. The base of a parallelogram measures 7 centimeters more than its height. If the area of the parallelogram is 30 square centimeters, find the measure of the base and height. *(Area = base * height)*



$x(x+7)=30$

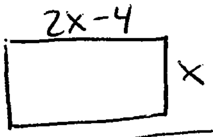
$x^2+7x-30=0$

$(x+10)(x-3)=0$

$\{\cancel{-10},+3\}$

$\boxed{\begin{matrix} x=3\text{ ft} \\ x+7=10\text{ ft} \end{matrix}}$

31. The length of a rectangle is 4 less than twice the width. The area of the rectangle is 70. Find the dimensions of the rectangle.



P=-140

S=-4

$x(2x-4)=70$

$2x^2-4x-70=0$

$\frac{(\cancel{2x-14})(2x+10)}{\cancel{2}}$

$(x-7)(2x+10)=0$

$\{7\ \cancel{-8}\}$

$\boxed{\begin{matrix} x=7\text{ ft} \\ 2x-4=10\text{ ft} \end{matrix}}$

32. What is the solution set of the equation $x^2-5x-24=0$?

[✓ (1) $\{-3,8\}$] (2) $\{-3,-8\}$ (3) $\{3,8\}$ (4) $\{3,-8\}$

$(x-8)(x+3)=0$

$\{8,-3\}$

33. Factored completely, the expression $2x^2 + 10x - 12$ is equivalent to

(1) $2(x-6)(x+1)$ [✓ (2)] $2(x+6)(x-1)$ (3) $2(x+2)(x+3)$ (4) $2(x-2)(x-3)$

$2(x^2+5x-6)=0$

$2(x+6)(x-1)$

34. What are the solutions of the equation $(y-2)(y-5)=0$?

(1) $y=-2$ and $y=5$ (2) $y=-2$ and $y=-5$ (3) $y=0$ and $y=2$ [✓ (4)] $y=2$ and $y=5$

35. Which of the following quadratic equations, in factored form, has the solution set $\{-3,5\}$?

(1) $(x-3)(x+5)=0$ (2) $3x(x-5)=0$ [✓ (3)] $(x+3)(x-5)=0$ (4) $5x(x+3)=0$

$(x+3)(x-5)$

36. Which equation has the solution set $\{1,3\}$?

[✓ (1)] $x^2-4x+3=0$ (2) $x^2-4x-3=0$ (3) $x^2+4x+3=0$ (4) $x^2+4x-3=0$

$(x-1)(x-3)=0$

$x^2-1x-3x+3=0$ x^2-4x+3

37. Greg is in a car at the top of a roller-coaster ride. The distance, d, of the car from the ground as the car descends is determined by the equation $d = 144 - 16t^2$, where t is the number of seconds it takes the car to travel down to each point on the ride. How many seconds will it take Greg to reach the ground?

$d = 144 - 16t^2$

$-16(t^2-9)$

$-16(t-3)(t+3)$

$\boxed{3}, -3$

REVIEW OF SEQUENCES

A sequence is an ordered list of numbers.

Arithmetic Sequence: when the pattern is ADDING

$$\underset{\nearrow\, n^{th}\text{ term}}{a_n} = \underset{\overset{\uparrow}{\text{first term}}}{a_1} + (\overset{\overset{\text{term position}}{\downarrow}}{n} - 1)\underset{\nwarrow\,\text{common difference}}{d}$$

Ex) Find the 100th term of: 3, 7, 11, 15, 19, …

Here: $n = 100$, $a_1 = 3$, $d = 4$ ➡ $a_{100} = 3 + (100-1)(4) = \boxed{399}$

Geometric Sequence: when the pattern is MULTIPLYING

$$a_n = a_1 \bullet r^{n-1}$$

a_1 is the first term of the sequence
r is the common ratio
n is the number of the term to find

Ex: Find the 7th term of the sequence: 2, 6, 18, 54, …

$n = 7$; $a_1 = 2$, $r = 3$ $\quad a_n = a_1 \bullet r^{n-1}$

$$a_7 = 2 \bullet 3^{7-1} = 1458$$

The seventh term is 1458.

Recursive Sequences: a term is found by knowing the term before it. The term "a_1" will be given along with a formula to find "a_n" given "a_{n-1}"

Ex) If $a_1 = 2$ and $a_n = 5a_{n-1} + 3$, find the first 4 terms.

Need to find a_1, a_2, a_3, and a_4

$a_1 = 2$

a_2 = plug in n to be 2 = $5a_{2-1} + 3 = 5a_1 + 3 = 5(2) + 3 = 13$

a_3 = plug in n to be 3 = $5a_{3-1} + 3 = 5a_2 + 3 = 5(13) + 3 = 68$

a_4 = plug in n to be 4 = $5a_{4-1} + 3 = 5a_3 + 3 = 5(68) + 3 = 343$

$$\boxed{2,\ 13,\ 68,\ 343}$$

Regents Review

1. What is the common difference of the arithmetic sequence 5, 8, 11, 14?

3 3 3

(1) $\frac{8}{5}$ (2) -3 [✓ (3) 3] (4) 9

2. What is the formula for the nth term of the sequence 54, 18, 6, ...?

$\frac{1}{3}$ $\frac{1}{3}$

(1) $a_n = 6\left(\frac{1}{3}\right)^n$

(3) $a_n = 54\left(\frac{1}{3}\right)^n$

(2) $a_n = 6\left(\frac{1}{3}\right)^{n-1}$

(4) $a_n = 54\left(\frac{1}{3}\right)^{n-1}$

$a_n = 54(\frac{1}{3})^{n-1}$

3. What is a formula for the nth term of sequence B shown below?

B = 10, 12, 14, 16, ...

2 2 2

[✓ (1) $b_n = 8 + 2n$]

(2) $b_n = 10 + 2n$

(3) $b_n = 10(2)^n$

(4) $b_n = 10(2)^{n-1}$

$10+(n-1)2$

$10+2n-2$

$8+2n$

4. A sequence has the following terms: $a_1 = 4$, $a_2 = 10$, $a_3 = 25$, $a_4 = 62.5$. Which formula represents the n^{th} term in the sequence?

$r = 2.5$

(1) $a_n = 4 + 2.5n$

(2) $a_n = 4 + 2.5(n-1)$

(3) $a_n = 4(2.5)^n$

[✓ (4) $a_n = 4(2.5)^{n-1}$]

$4(2.5)^{n-1}$

5. Find the first four terms of the recursive sequence defined below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

$a_1 = -3$

$a_2 = a(2-1) - 2$

$= a(1) - 2$

$= -3 - 2 = -5$

$a_3 = a(3-1) - 3$

$a(2) - 3$

$-5 - 3 = -8$

$a_4 = a(4-1) - 4$

$a(3) - 4$

$-8 - 4 = -12$

REVIEW OF STATISTICS

Measures of Central Tendency:

- Mean – average
- Median – the middle number (once the date is arranged in order). If there are two middle numbers, find the average of them.
- Mode – the number that appears the MOST often (there can be NO Mode or more than 1 mode)
- Range – difference between highest and lowest number.

Outlier – any number that is far away from the rest. When there are outliers, the median best represents the data.

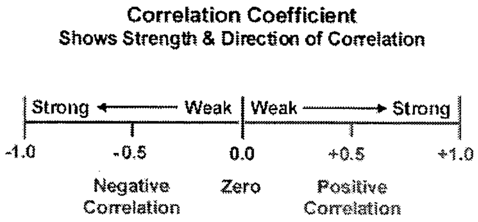
Quantitative – data is numbers
Qualitative – data isn't numbers (qualities)

Univariate – UNI = one set of numbers
Bivariate – BI = two sets of numbers

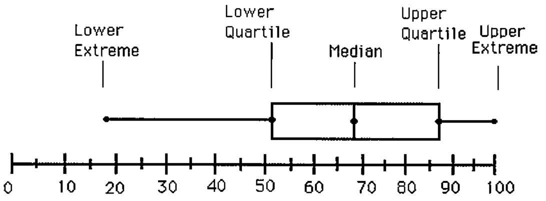
Causal Relationship – where one thing affects the other.

Correlation –

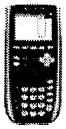
3 types POSITIVE – as one increases, the other increases
NEGATIVE – as one increases, the other decrease
NONE – scatter plot cannot be determined.



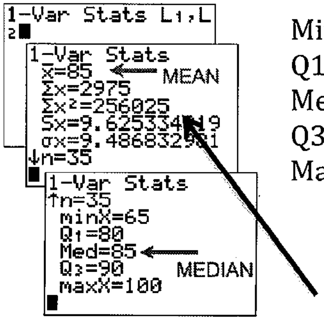
Box and Whisker Plot (Box Plot):



Interquartile range (IQR): difference between Q3 and Q1.



Using the Calculator:
Press STAT then EDIT
Enter data into L_1 and L_2
Press **STAT again.**
Arrow to the right to **CALC.**
Now choose option **#1: 1-Var Stats.**



```
1-Var Stats L₁,L
₂█
```

```
1-Var Stats
 x̄=85
 Σx=2975
 Σx²=256025
 Sx=9.625334[illegible]19
 σx=9.4868329[illegible]1
↓n=35
█
```

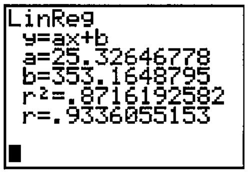
```
1-Var Stats
↑n=35
 minX=65
 Q₁=80
 Med=85
 Q₃=90
 maxX=100
█
```

Min – lowest extreme
Q1 – lower quartile
Med – Medial
Q3 – upper quartile
Max – upper extreme

Standard Deviation
Sx= ... (Sample standard deviation)

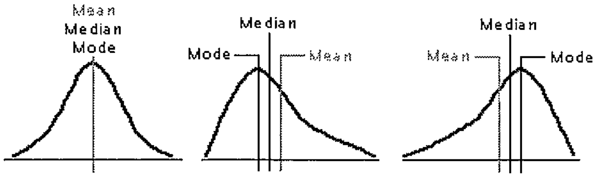
Linear Regression:

Press STAT, arrow right to CALC, and arrow down to 4: LinReg (ax+b). Hit ENTER.



```
LinReg
 y=ax+b
 a=25.32646778
 b=353.1648795
 r²=.8716192582
 r=.9336055153
```

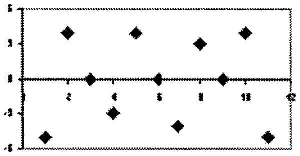
The correlation coefficient, *r*, is .9336055153 which places the correlation into the "strong" category. (0.8 or greater is a "strong" correlation)



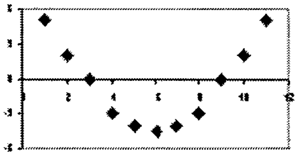
Symmetrical Distribution **Positive Skew (Skewed to the right)** **Negative Skew (Skewed to the left)**

- If symmetrical use the mean, if skewed use the median to describe the data

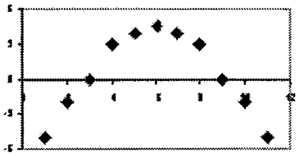
Residual Plot - is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data; otherwise, a non-linear model is more appropriate.



Random pattern



Non-random: U-shaped

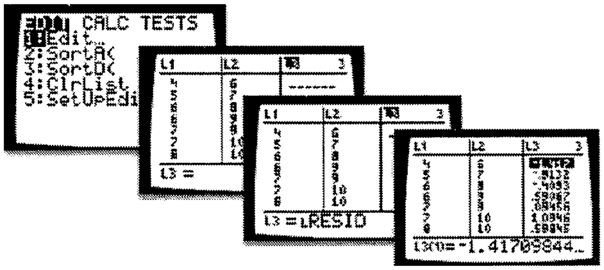


Non-random: Inverted U

Finding the Residual Plots

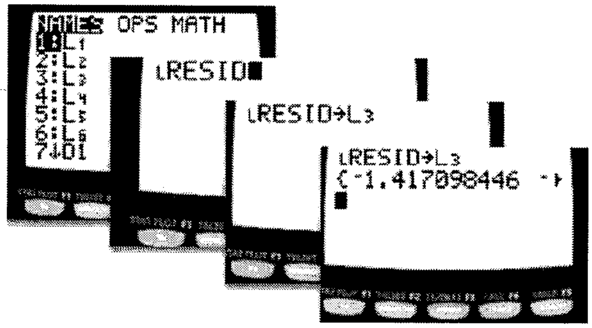
Method 1

Go to [Stat] "1: Edit". Select L3 with the arrow keys. [Enter] [2nd] "list". Scroll down and select RESID. [Enter] [Enter] again.



Method 2

Go to the main screen. [2nd] "list" [ENTER]. Scroll down and select RESID. [Enter]. [STO->] [2nd] "list". Select "3: L3" [ENTER].



```
ʟRESID
```

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ʟRESID→L3
```

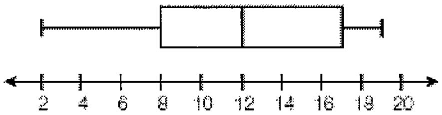
```
ʟRESID→L3
{⁻1.417098446 …
```

Regents Review

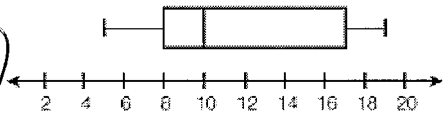
1.

The data set 5, 6, 7, 8, 9, 9, 9, 10, 12, 14, 17, 17, 18, 19, 19 represents the number of hours spent on the Internet in a week by students in a mathematics class. Which box-and-whisker plot represents the data?

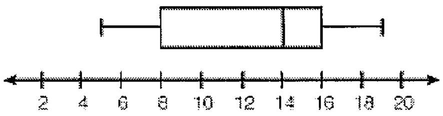
(1)



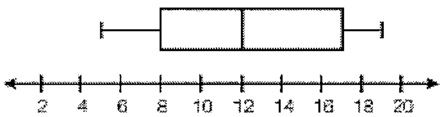
[✓ (2)]



(3)



(4)



2.

Which situation should be analyzed using bivariate data?

(1) Ms. Saleem keeps a list of the amount of time her daughter spends on her social studies homework.

[✓ (2)] Mr. Benjamin tries to see if his students' shoe sizes are directly related to their heights.

(3) Mr. DeStefan records his customers' best video game scores during the summer.

(4) Mr. Chan keeps track of his daughter's algebra grades for the quarter.

3.

What was the median high temperature in Middletown during the 7-day period shown in the table below?

Daily High Temperature in Middletown	
Day	Temperature (°F)
Sunday	~~68~~
Monday	~~73~~
Tuesday	~~73~~
Wednesday	75
Thursday	~~68~~
Friday	~~67~~
Saturday	~~69~~

67, 67, 68, 69, 73, 73, 75

[✓ (1) 69]
(2) 70
(3) 73
(4) 75

4.

Judy needs a mean (average) score of 86 on four tests to earn a midterm grade of B. If the mean of her scores for the first three tests was 83, what is the *lowest* score on a 100-point scale that she can receive on the fourth test to have a midterm grade of B?

$$\frac{83+83+83+X}{4} = 86$$

$$\boxed{X = 95}$$

5.

Melissa's test scores are 75, 83, and 75. Which statement is true about this set of data?

(1) mean < mode
(2) mode < median
[✓ (3) mode = median]
(4) mean = median

mod = 75
mean = 77.3
med = 75

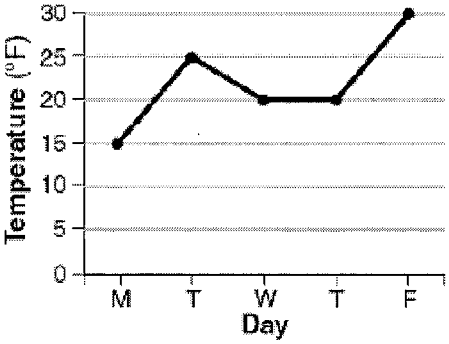
6.

Sara's test scores in mathematics were 64, 80, 88, 78, 60, 92, 84, 76, 86, 78, 72, and 90. Determine the mean, the median, and the mode of Sara's test scores.

$\bar{X} = 79$
med = 79
mode = 78

7.

The accompanying graph shows the high temperatures in Elmira, New York, for a 5-day period in January.



15
25
20
20
30

$\bar{X} = 22$
mode = 20
med = 20

Which statement describes the data?

[✓ (1) median = mode]
(2) median = mean
(3) mean < mode
(4) mean = mode

8.

From January 3 to January 7, Buffalo recorded the following daily high temperatures: ~~8~~5°, 7°, 6°, ~~8~~5°, and 7°. Which statement about the temperatures is true?

[✓ (1) mean = median]
(2) mean = mode
(3) median = mode
(4) mean < median

5, 5, 6, 7, 7
med = 6
mean = 6
mode = 7

9.

The accompanying table represents the number of cell phone minutes used for one week by 23 users.

Number of Minutes	Number of Users
71–80	10
61–70	7
51–60	2
41–50	3
31–40	1

23

11 11

12

Which interval contains the median?

(1) 41–50
(2) 51–60
[✓ (3) 61–70]
(4) 71–80

10.

Tamika could not remember her scores from five mathematics tests. She did remember that the mean (average) was exactly 80, the median was 81, and the mode was 88. If all her scores were integers with 100 the highest score possible and 0 the lowest score possible, what was the *lowest* score she could have received on any one test?

$$\frac{\underline{X} + \underline{80} + \underline{81} + \underline{88} + \underline{88}}{5} = 80$$

$$\boxed{X = 63}$$

11.

The students in Woodland High School's meteorology class measured the noon temperature every schoolday for a week. Their readings for the first 4 days were Monday, 56°; Tuesday, 72°; Wednesday, 67°; and Thursday, 61°. If the mean (average) temperature for the 5 days was exactly 63°, what was the temperature on Friday?

$$56, 72, 67, 61, \underline{\quad} = 63$$

$$\boxed{59}$$

12.

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

$$\bar{X} = 360 \qquad \boxed{350}$$

13.

The mean (average) weight of three dogs is 38 pounds. One of the dogs, Sparky, weighs 46 pounds. The other two dogs, Eddie and Sandy, have the same weight. Find Eddie's weight.

$$\frac{46 + 2X}{3} = 38$$

$$46 + 2X = 114$$

$$2X = 68$$

$$\boxed{X = 34}$$

14.

On an English examination, two students received scores of 90, five students received 85, seven students received 75, and one student received 55. The average score on this examination was

(1) 75 (3) 77

(2) 76 [✓ (4) 79]

2×90 1×55

5×85

7×75

$\frac{1185}{15} = 79$

15.

The exact average of a set of six test scores is 92. Five of these scores are 90, 98, 96, 94, and 85. What is the other test score?

(1) 92 [✓ (3) 89]

(2) 91 (4) 86

16.

Alex earned scores of 60, 74, 82, 87, 87, and 94 on his first six algebra tests. What is the relationship between the measures of central tendency of these scores?

(1) median < mode < mean (3) mode < median < mean

(2) mean < mode < median [✓ (4) mean < median < mode]

mean = 80.6

med = 84.5

Mode = 87

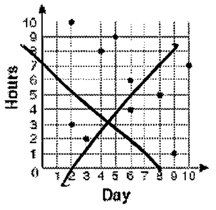
mean < med < mod

17.

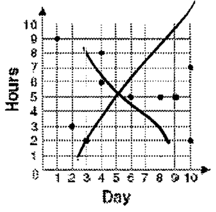
For 10 days, Romero kept a record of the number of hours he spent listening to music. The information is shown in the table below.

Day	1	2	3	4	5	6	7	8	9	10
Hours	9	3	2	6	8	6	10	4	5	2

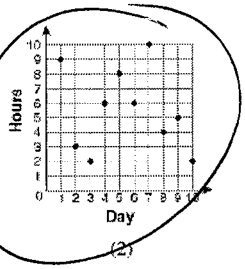
Which scatter plot shows Romero's data graphically?



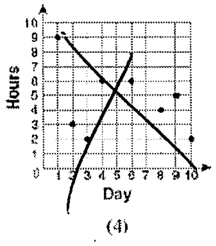
(1)



(3)



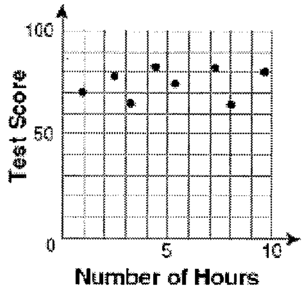
[✓ (2)]



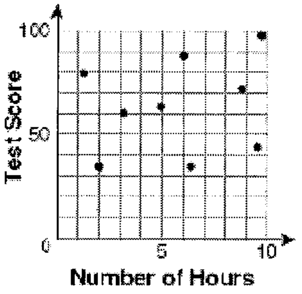
(4)

18.

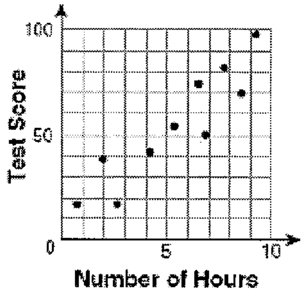
There is a negative correlation between the number of hours a student watches television and his or her social studies test score. Which scatter plot below displays this correlation?



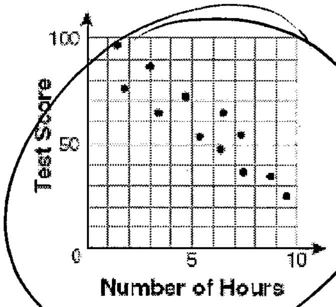
(1)



(3)



(2)



[✓ (4)]

19

Which data set describes a situation that could be classified as qualitative?

(1) the elevations of the five highest mountains in the world

(2) the ages of presidents at the time of their inauguration

[✓ (3) the opinions of students regarding school lunches]

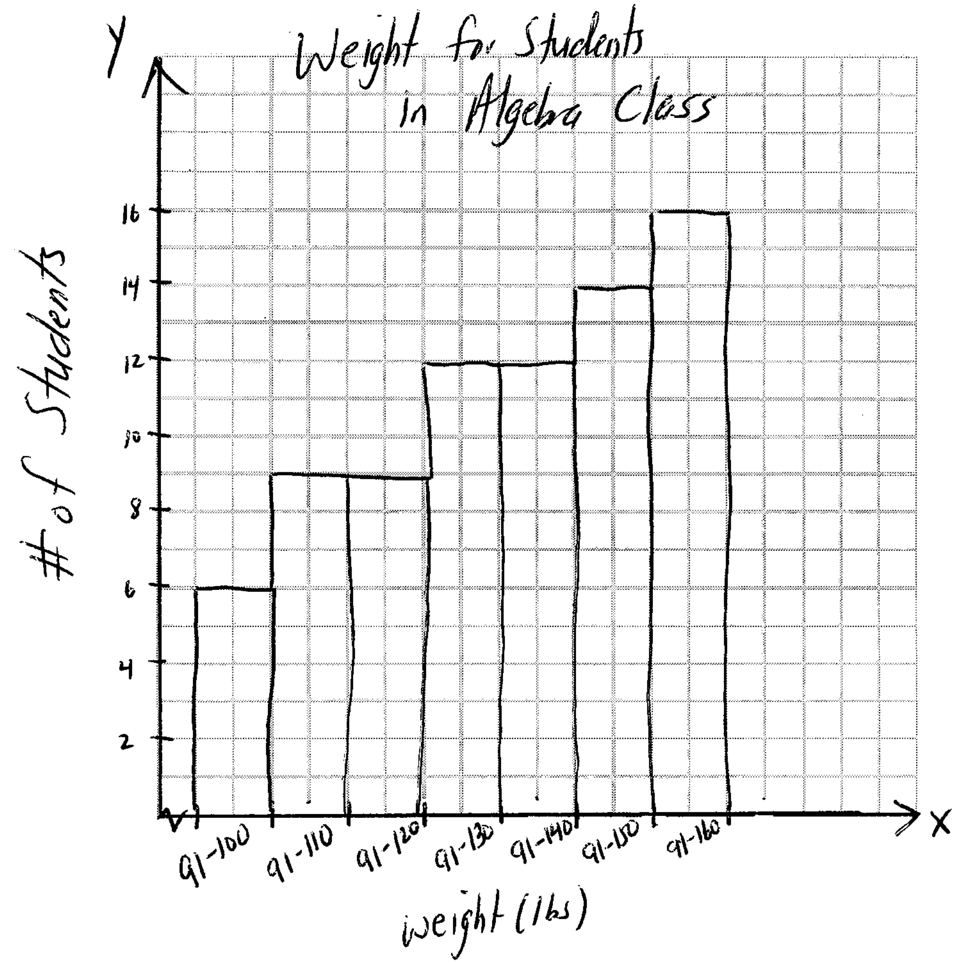
(4) the shoe sizes of players on the basketball team

20.

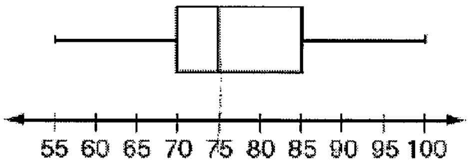
The accompanying table shows the weights, in pounds, for the students in an algebra class.

Using the data, complete the cumulative frequency table below and construct a cumulative frequency histogram on the grid on the next page.

Interval	Frequency	Cumulative Frequency
91–100	6	6
101–110	3	9
111–120	0	9
121–130	3	12
131–140	0	12
141–150	2	14
151–160	2	16



21. The accompanying box-and-whisker plot represents the scores earned on a science test.

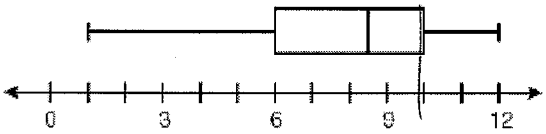


What is the median score?

(1) 70
[✓ (2) 75]
(3) 77
(4) 85

---

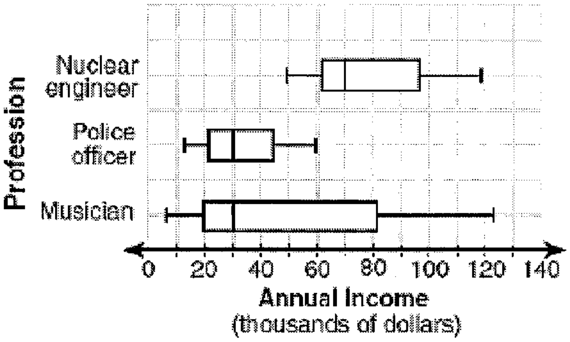
22. What is the value of the third quartile shown on the box-and-whisker plot below?



(1) 6
(2) 8.5
[✓ (3) 10]
(4) 12

---

23. The accompanying box-and-whisker plots can be used to compare the annual incomes of three professions.

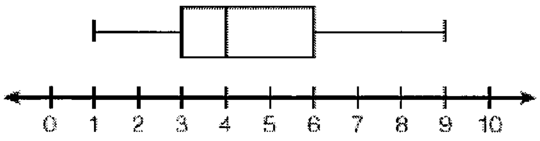


Based on the box-and-whisker plots, which statement is true?

(1) The median income for nuclear engineers is greater than the income of all musicians.
[✓ (2) The median income for police officers and musicians is the same.]
(3) All nuclear engineers earn more than all police officers.
(4) A musician will eventually earn more than a police officer.

24.

A movie theater recorded the number of tickets sold daily for a popular movie during the month of June. The box-and-whisker plot shown below represents the data for the number of tickets sold, in hundreds.



Which conclusion can be made using this plot?

~~(1)~~ The second quartile is 600.

~~(2)~~ The mean of the attendance is 400.

~~(3)~~ The range of the attendance is 300 to 600.

[✓ (4) Twenty-five percent of the attendance is between 300 and 400.]

25.

Which situation describes a correlation that is *not* a causal relationship?

[✓ (1) The rooster crows, and the Sun rises.]

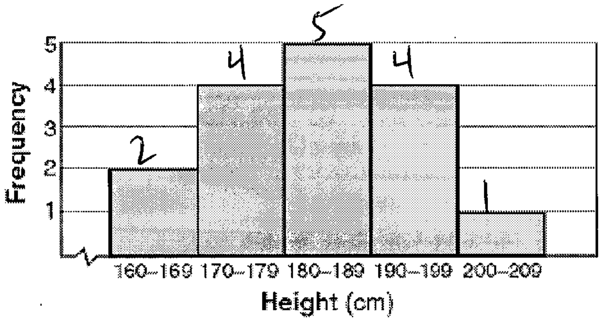
(2) The more miles driven, the more gasoline needed.

(3) The more powerful the microwave, the faster the food cooks.

(4) The faster the pace of a runner, the quicker the runner finishes.

26.

The accompanying histogram shows the heights of the students in Kyra's health class.



What is the total number of students in the class?

(1) 5

(2) 15

[✓ (3) 16]

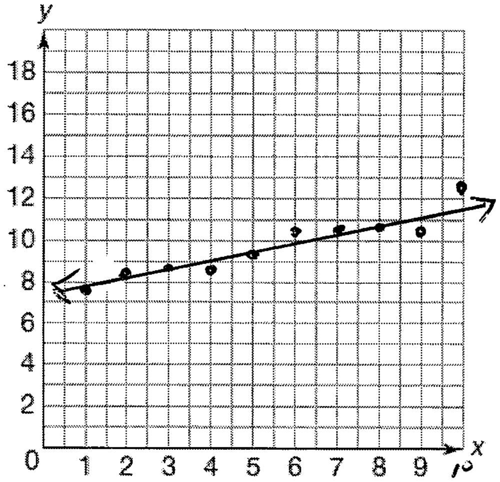
(4) 209

27. The table shows the percent of the United States population who did not receive needed dental care services due to cost.

	1	2	3	4	5	6	7	8	9	10	11
Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Percent	7.9	8.1	8.7	8.6	9.2	10.7	10.7	10.8	10.5	12.6	13.3
residual	.36	.05	.14	-.48	-.39	.6	.09	-.32	-1.1	-.45	.64

$\boxed{2nd}\ \boxed{STAT}$

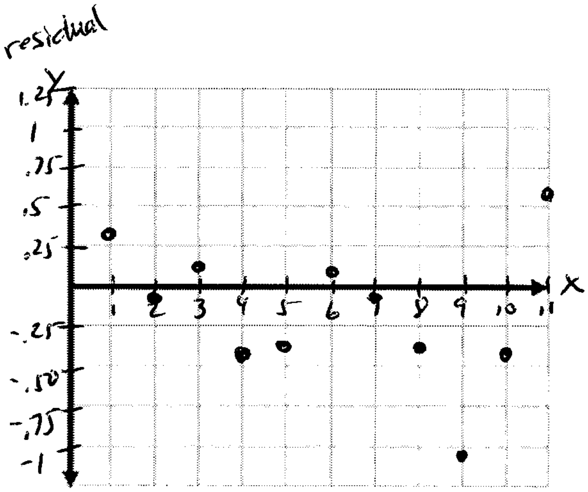
a. Sketch a scatter plot of the data



b. Using two point from ~~the data estimate~~ the equation of the line of best fit.

$y = .51x - 7.03$

c. Using the estimated line of best fit equation, calculate the residuals for the set of data (round to one decimal place). Construct a residual plot for the data.



Review of Word Problems

Always start with a "LET" statement to define your variable(s)

Consecutive Integer Problems:	*Consecutive*	*Consecutive Even/Odd*
	Let $x = 1^{st}$	*Let* $x = 1^{st}$
	$x + 1 = 2^{nd}$	$x + 2 = 2^{nd}$
	$x + 2 = 3^{rd}$	$x + 4 = 3^{rd}$

Perimeter Word Problems

The perimeter of a rectangle is 104m. The length is 7m more than twice the width. What are the dimensions of the rectangle?

Let:
x = width
$2x + 7$ = length

Perimeter of a rectangle = (2 x length) + (2 x width)

$2x + 2(2x + 7) = 104$ {perimeter = 2(length + 2(width)}
$2x + 4x + 14 = 104$ {used distributive property}
$6x + 14 = 104$ {combined like terms}
$6x = 90$ {subtracted 14 from both sides}
$x = 15$ {divided both sides by 6}

$2x + 7 = 37$ {substituted 15, in for x, into $2x + 7$}

width = 15 m and length = 37 m

Area Word Problems

The length of a rectangle equals twice its width and its area is $32m^2$. Find the dimensions of this rectangle.

Let w = width
$2w$ = length

$w(2w) = 32$

$2w^2 = 32$
$w^2 = 16$
$w = 4$

The length equals $2(4) = 8$

The dimensions of the rectangle are 8m by 4m.

Algebra 1 Vocabulary

absolute value function a function in which the input is contained within absolute value symbols

accuracy how close a measurement or calculation is to its actual value

additive identity the number that, when added to a number a, gives the sum a; for real numbers, the additive identity is 0: $a + 0 = a$

additive inverse for any real number a, the number $-a$, such that their sum is the additive identity: $a + (-a) = (-a) + a = 0$

approximation a value used to represent a true measurement when an exact answer is not possible

arithmetic sequence a sequence in which successive terms have a common difference

asymptote a line that the graph of a function continuously approaches but never touches

axis of symmetry (of a parabola) a vertical line of symmetry passing through the vertex of a parabola

base the number or variable that is raised to a power in an exponential expression

bimodal distribution a distribution of data that, when graphed, shows two clear peaks

binomial a polynomial containing exactly two unlike terms

bivariate data statistical data in which two variables are being studied

box plot a graph above a number line that shows the lower and upper extremes, first and third quartiles, and median of a data set; also called a box-and-whisker plot

categorical data data that cannot be measured and are generally in the form of names or labels

ceiling function See **least integer function**.

coefficient a number that is multiplied by a variable in an expression or equation

common difference the number added to find the next term in an arithmetic sequence

common ratio the number by which each term in a geometric sequence is multiplied to obtain the next term

completing the square a method of converting a quadratic expression of the form $ax^2 + bx + c$ to the form $a(x - h)^2 + k$

compound inequality an inequality that has two or more boundaries

conditional frequency a relative frequency in the body of a two-way relative frequency table

constant a number with a known value that does not change in a mathematical expression

continuous not having any jumps or breaks in shape; able to be drawn in one motion without interruption

conversion factor a number used to convert from one unit to another through multiplication or division

correlation coefficient a number r, where $-1 \leq r \leq 1$, that describes the strength of the association between two variables

curve of best fit the curve that most closely represents the relationship between variables that do not have a linear association

degree (of a polynomial) a characteristic of a polynomial determined by the highest exponent or sum of exponents of any term

dependent variable a variable, often y or $f(x)$, that provides the output value of an equation or function

dimensional analysis a method of determining or checking a mathematical expression for a given context by examining units

discriminant the radicand expression, $b^2 - 4ac$, from the quadratic formula, which can be used to determine how many real roots a quadratic equation has

domain the set of all the first elements (inputs) of a relation

dot plot a data display that represents data values as dots over a number line

element an individual value from a set

elimination method a method for solving systems of equations where equations are multiplied by constants and added and/or subtracted so as to eliminate all but one variable

end behavior the behavior of a graph as it is followed farther and farther in either direction

estimate a value made inexact on purpose in order to make calculations easier or to generalize about a population

even function a function that is symmetrical with respect to the y-axis

experimental study a study in which the researcher controls variables in order to determine their effect

exponent the number in an exponential expression that indicates how many times a base is multiplied by itself

exponential decay a relationship modeled by a function of the form $f(x) = a \cdot b^x$ in which $a > 0$ and $0 < b < 1$

exponential equation an equation in which the variable is in the exponent

exponential function a function of the form $f(x) = a \cdot b^x + c$, in which the input, x, is the exponent of a constant, b

exponential growth a relationship modeled by a function of the form $f(x) = a \cdot b^x$ in which $a > 0$ and $b > 1$

extraneous solution a value of a variable that is obtained by solving an equation but that is not a solution to the equation or to the situation that the equation models

first quartile (Q_1) the median of the lower half of a data set

floor function See **greatest integer function.**

function a relation in which every input is assigned to exactly one output

geometric sequence a sequence in which consecutive terms have a common ratio

greatest integer function a step function that outputs the greatest integer that is less than or equal to the input; also called a floor function

half-plane the portion of the coordinate plane that lies on one side of a line

histogram a data display that uses bars to show how frequently data occur within certain ranges or intervals

horizontal line test a test in which if any horizontal line crosses a graph of a relation at two or more points, then the inverse of that relation is not itself a function

horizontal shrink a transformation that pushes the points of a figure or graph toward the y-axis

horizontal stretch a transformation that pulls the points of a figure or graph away from the y-axis

horizontal translation a slide of a graph or figure in the right or the left direction on the coordinate plane

independent variable a variable, often x, that serves as the input value of an equation or function

index a small number indicating what root is being taken in a radical expression

input the first value, often an x-coordinate, in an ordered pair for a function; the value that is entered into a function in order to produce the related output

interquartile range (IQR) a measure of the spread of the middle 50% of a data set; equal to the difference of the first and third quartiles of the set

inverse (of a function) the relation that swaps the input and output of a given function

irrational number a number that cannot be written as a quotient of integers

joint frequency a frequency in the body of a two-way frequency table

leading coefficient (of a quadratic equation) the coefficient a of a quadratic equation in standard form, $y = ax^2 + bx + c$

least integer function a step function that outputs the least integer that is greater than or equal to the input; also called a ceiling function

linear equation an equation in which every variable is raised to the first power

linear function a function of the form $f(x) = mx + b$, in which the input, x, is raised to the first power and whose graph is a straight line

line of best fit the line that most closely represents the relationship between variables that have a linear association; also called a trend line

line of reflection the line over which a figure or graph is flipped to produce a mirror image

lower extreme the least value in a data set

marginal frequency an entry in the "Total" row or "Total" column of a two-way frequency table or a two-way relative frequency table

maximum the point on a graph that has the greatest y- or $f(x)$-value

mean the sum of all the terms in a data set divided by the total number of terms

measure of center a value that represents the middle or average of a data set

median the middle value in a data set that is ordered from least to greatest

minimum the point on a graph that has the least y- or $f(x)$-value

monomial a polynomial containing only one term

multiplicative identity the number which, when multiplied by a number a, gives the product a; for real numbers, the multiplicative identity is 1: $a \times 1 = a$

multiplicative inverse for any real number a other than 0, the number $\frac{1}{a}$ such that their product is the multiplicative identity: $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$

normal distribution a distribution of data which, when graphed, is symmetrical and resembles a bell curve

observational study a study in which variables are observed or outcomes are measured, but no attempt is made to control variables or affect outcomes

odd function a function that is symmetrical with respect to the origin

outlier an element that is very different from the other elements in the same data set

output the second value, often a y-coordinate, in an ordered pair for a function; the value that is produced when a function is evaluated for a given input

parabola the U-shaped graph of a quadratic function

parent function the most basic function in a family, or group, of related functions

piecewise function a function in which the output is calculated according to two or more rules, depending on the input

polynomial a collection of constants and variables joined through addition, subtraction, and multiplication

power the exponent in an exponential expression; the number that indicates how many times a base is used as a factor

prime factorization a string of prime factors whose product is a given number or polynomial

prime number a positive integer that cannot be divided without remainder by any positive integer other than itself and 1

principal square root the positive square root of a number

quadratic expression a polynomial expression of degree 2

quadratic formula the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ used to find the solutions to a quadratic equation of the form $ax^2 + bx + c = 0$

quadratic function a function in which the highest power of the variable is 2

quantitative data data that can be measured and are in numerical form

radical an expression of the form $\sqrt{r}$ or $\sqrt[n]{r}$, where r is a number or expression

radicand the number or expression inside a radical ($\sqrt{}$) sign

range (of a function) the set of all the second elements (outputs) in a relation

rate of change the value by which one quantity changes when another related quantity increases by a unit amount

rational exponent in an exponential expression, an exponent that is a rational number

rational number a number that can be written as a quotient of integers, $\frac{a}{b}$

reciprocal the multiplicative inverse of a number

recursive process a process that requires knowing or computing previous terms in order to find the value of a desired term

reflection a transformation that flips a figure or graph over a point or line

relation a set of ordered pairs

relative frequency the ratio of a frequency for a category to the total frequencies in a row, a column, or an entire table

residual the difference of an observed y-value on a scatter plot and a predicted y-value, based on a line of fit

root a factor of a number that, when multiplied by itself a given number of times, equals the number

scatter plot a graph that shows the relationship between two variables; a graph on which data are plotted as points (x, y) on a coordinate plane

sequence a predictable arrangement of numbers, expressions, pictures, or other objects that follows a pattern or rule

skewed distribution a distribution of data which, when graphed, shows a "tail" that extends much more to one side of the graph than to the other

slope the ratio of the vertical change to the horizontal change for the graph of a linear equation

slope-intercept form a form of a linear equation, $y = mx + b$, where m is the slope and b is the y-intercept of the graph

spread (of a data set) describes how data in a given data set are distributed or grouped

standard deviation a measure of spread for a set of data that indicates how much a data set varies from the mean

standard form (of a quadratic equation) the form $y = ax^2 + bx + c$ of a quadratic equation in which a, b, and c are constants

standard form (of a quadratic function) the form $f(x) = ax^2 + bx + c$ of a quadratic function

step function a piecewise function in which each interval has a constant value and which forms a graph made up of "steps"

substitution method a method for solving systems of equations where one variable is replaced by an equivalent expression in the other variable

system of linear equations a grouping of two or more linear equations written using the same variables

tangent intersecting a curve at only one point

term (of an expression) a combination of constants and/or variables joined together through multiplication or division

term (of a sequence) a number, expression, picture, or other object that is part of a sequence

third quartile (Q_3) the median of the upper half of a data set

transformation an operation that changes a figure or graph according to a rule

translation a transformation that moves all of the points on a figure the same distance in the same direction

trinomial a polynomial containing exactly three unlike terms

two-way frequency table a data display used to display and interpret frequencies for categorical variables

two-way relative frequency table a data display used to display and interpret relative frequencies for categorical variables

uniform distribution a distribution of data in which all values have the same frequency

upper extreme the greatest value in a data set

variable a letter or symbol that represents an unknown or changing number in a mathematical expression

vertex the turning point for the graph of a quadratic or absolute value function

vertex form (of a quadratic equation) the form $y = a(x - h)^2 + k$ of a quadratic equation in which (h, k) is the vertex

vertical line test test in which if any vertical line crosses a graph at two or more points, then the graph does not represent a function

vertical shrink a transformation that pushes the points of a figure or graph toward the *x*-axis

vertical stretch a transformation that pulls the points of a figure or graph away from the *x*-axis

vertical translation a slide of a graph or figure up or down on the coordinate plane (Lesson 18)

***x*-intercept** a point $(a, 0)$ at which a graph crosses the *x*-axis

***y*-intercept** a point $(0, b)$ at which a graph crosses the y-axis

zero (of a function) an input value for a function that produces 0 as the output; equal to the x-coordinate of an x-intercept of the function

zero product property property stating that if the product of two numbers or expressions is equal to 0, then one of those numbers or expressions must be equal to 0