Common Core Algebra Regents Review

<u>Real numbers, properties, and operations</u>:

1) The set of <u>natural</u> numbers is the set of counting numbers. $\{1,2,3,...\}$ symbol $\mathbb{N}$

2) The set of <u>whole</u> numbers is the set of all the natural numbers and zero. $\{0,1,2,3,...\}$ (no symbol)

3) The set of <u>integers</u> is the set of whole numbers and their opposites. $\{...,-2,-1,0,1,2,...\}$ symbol $\mathbb{Z}$

4) The set of <u>rational</u> numbers is the set of all numbers that can expressed as fractions in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero. symbol $\mathbb{Q}$

5) The set of <u>irrational</u> numbers is the set of all non-repeating, non-terminating decimals. (no symbol) An irrational number cannot be expressed as an integer or the quotient of integers.

6) The set of <u>real</u> numbers is the union of the rational and irrational numbers.

7) The absolute value of a number is its measured distance from zero on a number line.

8) Scientific notation is used to represent very large or very small numbers.

9) A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10. ($3.1\times10^5 = 310{,}000$) Small numbers have negative exponents, large numbers have positive exponents.

10) A power of a number represents repeated multiplication of the number. For example, $(-5)^4$ means $(-5)\cdot(-5)\cdot(-5)\cdot(-5)$. The base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. In this example, (–5) is the base, and 4 is the exponent. IMPORTANT NOTE: You MUST use ( ) around a negative number when you raise it to a power, otherwise you will get the wrong answer. $-5^4 = -625$, while $(-5)^4 = 625$.

11) Any real number raised to the zero power is 1. The only exception to this rule is zero itself. 0^0 is an indeterminate form and cannot be evaluated. (you don't need to know that for the regents.)

12) A perfect square is a whole number whose square root is a whole number ($\sqrt{25} = 5$; thus, 25 is a perfect square).

13) The square root of a number is a number which when multiplied by itself equals the number under the radical. The number under the radical is called the radicand.

14) Any whole number other than a perfect square has a square root that lies between two consecutive whole numbers.

15) The square root of a whole number that is not a perfect square is an irrational number ($\sqrt{2}$ is an irrational number). An irrational number cannot be expressed exactly as a ratio.

16) A set is closed under a binary operation when every pair of elements from the set, under the given operation, yields and element from that set.

17) Commutative Properties of Addition and Multiplication: $x + y = y + x$ & $x \bullet y = y \bullet x$.

18) Associative Properties of Addition and Multiplication: $(x+y)+z=x+(y+z)$ & $(xy)z=x(yz)$.

19) Distributive Property of Multiplication over Addition: $x(y+z)=xy+xz$.

20) Additive Identity Property: $a+0=0+a=a$.

21) Additive Inverse Property: $a+-a=-a+a=0$

22) Multiplicative Identity Property: $a\bullet 0=0\bullet a=a$

23) Multiplicative Inverse Property: $a\bullet\frac{1}{a}=\frac{1}{a}\bullet a=1$

24) Zero Product Property: If $a\bullet b=0$, then $a=0$, or $b=0$, or both a and b equal 0. We use this when solving quadratic equations.

25) To simplify a radical that is not a perfect square:
1) Find the factors of the radicand, one of which should be a perfect square.
2) Find the square root of this factor, leaving the remaining factor under the radical.
3) Continue until there are no more perfect squares.

Questions to Try:

1) Which is an irrational number?

(1) $\sqrt{9}$ (2) 3.14 (3) $\sqrt{3}$ (4) $\frac{3}{17}$

2) Which value is rational?

(1) π (2) $\sqrt{\frac{1}{2}}$ (3) $\frac{5}{13}$ (4) $\sqrt{17}$

3) The expression $3^2\bullet 3^3\bullet 3^4$ is equivalent to:
(1) 27^9 (2) 27^{24} (3) 3^9 (4) 3^{24}

4) The statement "n is even and a perfect square" is true when n equals:
(1) 1 (2) 18 (3) 25 (4) 4

5) Which property is illustrated by the equation $ax+ay=a(x+y)$?

(1) associative (2) commutative (3) distributive (4) identity

6) Which is the multiplicative inverse of 5?

(1) 1 (2) $\frac{1}{5}$ (3) $-\frac{1}{5}$ (4) -5

7) Which is the additive inverse of 5?

(1) 1 (2) $\frac{1}{5}$ (3) $-\frac{1}{5}$ (4) -5

8) Under which operation is the set $\{-1,0,1\}$ closed?

(1) multiplication (2) division (3) addition (4) subtraction

9) Expressed in scientific notation the number 4,600,000,000 is
(1) 4.6×10^{-8} (2) 4.6×10^{-8} (3) 4.6×10^{9} (4) 0.46×10^{10}

10) If $r = 2$ and $n = -7$, what is the value of $|r| - |n|$?
(1) 5 (2) 9 (3) -5 (4) -9

11) The expression $\sqrt{50}$ can be simplified into
(1) $2\sqrt{25}$ (2) $5\sqrt{10}$ (3) $25\sqrt{2}$ (4) $5\sqrt{2}$

12) Express $4\sqrt{60}$ in simplest radical form.

13) Theo determined that the correct length of the hypotenuse of a right triangle whose legs are 2 and 4 is $\sqrt{20}$. Fiona says the length of the hypotenuse is $2\sqrt{5}$. Is Fiona correct? Justify your answer.

14) What is the value of the expression $-x^2y+4x$ when $x=-3$ and $y=4$?
(1) 24 (2) -20 (3) 52 (4) 20

Expressions, Equations and Inequalities:

1) Algebraic expressions are evaluated by replacing the variables with numbers and applying the order of operations to simplify the resulting expression.
2) An algebraic equation is a mathematical statement that says that two expressions are equal.
3) An inequality is a mathematical sentence that states that one quantity is less than (or greater than) another quantity.
4) To maintain equality, an operation that is performed on one side of an equation must be performed on the other side.
5) When both expressions of an inequality are multiplied or divided BY a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$).
6) An equation involving two or more variables is called a literal equation. In this type of equation, we want to solve for one variable in terms the other variables.
7) Key words in translating verbal expressions/sentences to algebraic expressions/equations may include words and their translations such as: *is* to =, *of* to *multiplication*, *more than* to +, *less than* to –, *increased by* to +, and *decreased by* to –.
8) Word Problems: Define each unknown (variable) in a "Let statement". Express as many of the unknowns in terms of your variable, or use a second variable.
9) Consecutive Integers are integers that follow one another in order: 1,2,3,... and are represented as x, x + 1, x + 2,...
10) Consecutive even integers are **even** integers that follow one another in order: 0,2,4,... and are represented as x, x + 2, x + 4,...

11) Consecutive odd integers are **odd** integers that follow one another in order: 1,3,5,... and are represented as x, x + 2, x + 4,... No, that is not a typo. Represented the same way as evens.

Questions to try:

1) Tara bought two items that cost d dollars each. She gave the cashier \$20. Which expression represents the change she received?
(1) $20-2d$ (2) $20-d$ (3) $20+2d$ (4) $2d-20$

2) Which expression represents "5 less than the product of 7 and x"?
(1) $7(x-5)$ (2) $5-7x$ (3) $7x-5$ (4) $5-7x$

3) The sum of Scott's age and Greg's age is 33 years. If Greg's age is represented by g, Scott's age is represented by:

(1) $33-g$ (2) $g-33$ (3) $g+33$ (4) 33g

4) Which equation can be used to solve the problem below?
If four times a number is increased by 15, the result is three less than six times the number. Find the number.
(1) $4(x+15)=6x-3$ (2) $4x+15=6(x-3)$ (3) $4x+15=6x-3$ (4) $4x+15=3-6x$

5) Mario paid \$42.50 in taxi fare for his ride from the hotel to the airport. The cab charged \$2.25 for the first mile plus \$3.50 for each additional mile. How many miles was it from the hotel to the airport?

6) When Albert flips open his math textbook, he notices that the product of the page numbers of the two facing pages he sees is 156. Find, algebraically, the page numbers that Albert is looking at.

7) When solved for y, the equation $ay-b=c$ is equivalent to:

(1) $\frac{c-b}{a}$ (2) $\frac{c+b}{a}$ (3) $\frac{c+b}{y}$ (4) $\frac{c+a}{b}$

8) If $x=2a-b^2$, then a equals

(1) $\frac{x-b^2}{2}$ (2) $\frac{x+b^2}{2}$ (3) $\frac{b^2-x}{2}$ (4) $x+b^2$

9) The equation $P = 2L + 2W$ is equivalent to:

(1) $L = \frac{P - 2W}{2}$ (2) $L = \frac{P + 2W}{2}$ (3) $2L = \frac{P}{2W}$ (4) $L = P - W$

10) In the equation $A = p + prt$, t is equivalent to:

(1) $\frac{A - pr}{p}$ (2) $\frac{A - p}{pr}$ (3) $\frac{A}{pr} - p$ (4) $\frac{A}{p} - pr$

11) If $2n + 1$ represents an odd integer, the next larger odd integer is represented by

(1) $2n+3$ (2) $2n+2$ (3) $2n$ (4) $2n-1$

12) In the Ambrose family, the ages of the three children are consecutive even integers. If the age of the youngest child is represented by $x+3$, which expression represents the age of the oldest child?

(1) $x+5$ (2) $x+6$ (3) $x+7$ (4) $x+8$

13) a) An electronics store sells DVD players and cordless telephones. The store makes a \$75 profit on the sale of each DVD player, *d,* and a \$30 profit on each sale of a cordless phone, *c.* The store wants to make a profit of at least \$550 from its sales of these 2 items. Write an inequality that describes this situation.

b) If the store sold 5 DVD players, what is the least number of cordless phones they need to sell in order to make the profit they want?

14) Find the value of x in the equation $13 - 2(x+4) = 8x + 1$.

15) A truck traveling at a constant rate of 45 miles per hour leaves Albany. One hour later a car traveling at a constant rate of 60 miles per hour also leaves Albany traveling in the same direction on the same highway. How long will it take for the car to catch up to the truck if both vehicles continue in the same direction on the highway?

16) Rhonda has $1.35 in nickels and dimes in her pocket. If she has six more dimes than nickels, which equation can be used to determine x, the number of nickels she has?

(1) $0.05(x+6)+0.10x=1.35$ (3) $0.05+0.10(6x)=1.35$

(2) $0.05x+0.10(x+6)=1.35$ (4) $0.15(x+6)=1.35$

<u>Sets</u>:

1) A <u>set</u> is a collection of objects or elements, such as a set of numbers. The universal set is the set of all elements in a given situation.

2) The intersection of two sets *A* and *B*, denoted by $A \cap B$, is the set of all elements that belong to both sets *A* and *B*. Two sets are disjoint if there intersection is the empty set. Only list the common elements when stating the intersection.

3) The union of two sets, *A* and *B*, denoted $A \cup B$, is the set of all elements that belong to set *A* or set *B*, or to both set *A* and set *B*. List all the elements from both sets when stating the union.

4) The complement of a set A, denoted $\overline{\text{A}}$ is the set of all elements that belong to the universe, *U* but do not belong to set *A*.

5) Interval notation is used when we do not want to state a variable when describing a set. It is also used when we are describing an interval on a number line. Use square braces [or] if the number is included, and rounded parenthesis ( ) if the number is not included. ∞ and $-\infty$ always have (or) next to them.

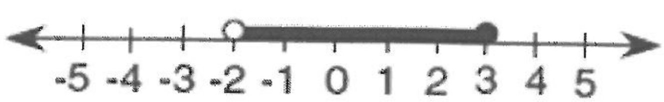
Set	Interval	Set	Interval	Set	Interval
$x<3$	$(-\infty,3)$	$x>3$	$(3,\infty)$	$-3\le x<2$	$[-3,2)$
$x\le 3$	$(-\infty,3]$	$x\ge 3$	$[3,\infty)$	$-3<x\le 2$	$(-3,2]$

6) Set builder notation $\{x \in \mathbb{R} \mid (x<2) \vee (x\ge 5)\}$ means the set x, where x is an element of the real numbers, such that x is less than 2 or x is greater than or equal to 5.

7) $\vee$ means OR, and $\wedge$ means AND. When asked to graph a solution set on a number line, if you see $\vee$ plot all values that fit the conditions. If you see $\wedge$ only plot the values that fit both conditions.

<u>Questions to try</u>:

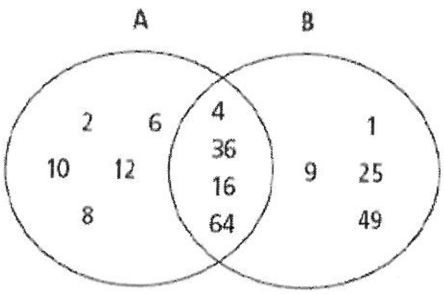
1) a) Which inequality is represented by the graph below?



(1) $-2\le x\le 3$ (2) $-2<x<3$ (3) $-2\le x<3$ (4) $-2<x\le 3$

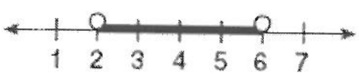
b) Express each of the choice in part *a* using interval notation.

2) Consider the following diagram and answer the questions.



1. List the elements of set A
2. List the elements of set B
3. List the elements of the Universal set, U
4. List the elements of $A \cap B$
5. List the elements of $A \cup B$
6. List the elements of $\overline{A}$
7. List the elements of $\overline{B}$

3) Which graph represents the solution set for $2x - 4 > 8$ or $x + 5 < 7$

(1)


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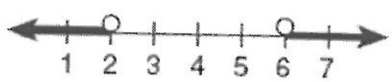

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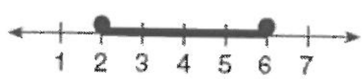
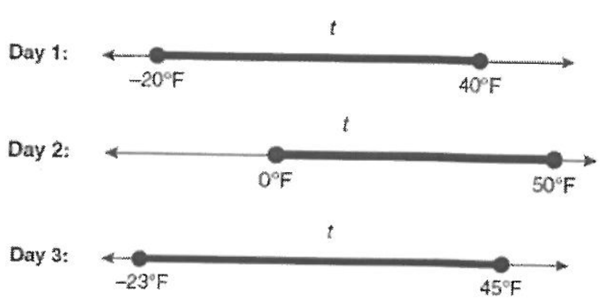


(4)


4) Maureen tracked the range of outdoor temperatures over three days. She recorded the following information:



Express the intersection of the 3 sets of data as an inequality in terms of temperature.

5) Consider the set of integers greater than -2 and less than 6. A subset of this set is the positive factors of 5. What is the complement of this subset?

(1) $\{0,2,3,4\}$ (2) $\{-1,0,2,3,4\}$ (3) $\{-2,-1,0,2,3,4,6\}$ (4) $\{-2,-1,0,2,3,4,5,6\}$

6) Given: Set A = $\{(-2,-1),(-1,0),(1,8)\}$ Set B = $\{(-3,-4),(-2,-1),(-1,2),(1,8)\}$

a) Find $A \cap B$.

b) Find $A \cup B$.

c) Find the domain of Set A.

d) Find the range of Set B.

7) If the universal set $U = \{1,2,3,...,20\}$, $M = \{1,3,6,9,12,15,18\}$, and $N = \{2,4,6,...,20\}$, what is $M \cap \overline{N}$?

8) Which set describes the empty set?

(1) $\{x \mid (x>2) \vee (x<-4)\}$ (3) $\{x \mid x \in \mathbb{N} \text{ and } x \geq -2\}$

(2) $\{x \mid (x>2) \wedge (x<-4)\}$ (4) $\{x \mid x \in \mathbb{Z} \text{ and } x > -5\}$

9) Which sentence best describes the following set? $\{-2x-1 \mid x \in \text{ the set of whole numbers}\}$

(1) The set of integers less than 0.
(2) The set of integers one less than twice as big as x.
(3) The set of natural numbers greater than one less than twice a whole number.
(4) The set of negative odd integers.

10) Find the solution set for the open sentence $5x \leq 16$, using the domain = $\{\textit{odd whole numbers}\}$.

(1) $\{1,2,3\}$ (2) $\varnothing$ (3) $\{1,3\}$ (4) $\{3.2\}$

11) The cost, y, for a pizza from King's is \$4 plus \$0.50 for each topping, x. Which of the following sets best describes the total cost for one pizza?

(1) $\{x \mid y = 4x + 0.5 \text{ and } x \in \text{ the set of integers}\}$

(2) $\{x \mid x \text{ is an integer and } x > 4.50\}$

(3) $\{y \mid y = 0.5x + 4 \text{ and } x \in \text{ the set of whole numbers}\}$

(4) $\{y \mid y \text{ is a whole number and } x = 4 + 0.5y\}$

Factoring

1) The GCF is the largest monomial factor of a set of terms. Always factor out a GCF if there is one.

2) After performing step 1, you will encounter 4 possible scenarios:

a) There is a binomial that is the difference of 2 perfect squares. Place the square root of each term in 2 different parentheses and use one + and one -.

$4x^2 - 9 \rightarrow \sqrt{4x^2} = 2x$ and $\sqrt{9} = 3$ Factors: $(2x+3)(2x-3)$

b) There is a trinomial whose leading coefficient is 1.
Find factors of the last term of the polynomial that sum to the middle term's coefficient. If the sign in front of the third term is +, the binomials will have the same sign, either both positive or both negative. The sign in front of the second term will decide which sign. If the sign in front of the third term is −, the binomials will have different signs, and the sign in front of the second term will be the sign of the larger factor.

c) There is a trinomial whose leading coefficient is > 1.

Example Given:	$5x^2 + 11x + 2$
Find the product ac:	$(5)(2) = 10$
Think of two factors of 10 that add up to 11:	1 and 10
Write the 11x as the sum of 1x and 10x:	$5x^2 + 1x + 10x + 2$
Group the two pairs of terms:	$(5x^2 + 1x) + (10x + 2)$
Remove common factors from each group:	$x(5x+1) + 2(5x+1)$
Factor out a common factor of $(5x+1)$	$(5x+1)(x+2)$

d) None of a-c applies. It was only a GCF problem.

Factor each by grouping:

1) $7x^2 + 22x + 3$

2) $2x^2 + 15x + 18$

3) $2x^2 - 3x - 5$

4) $3x^2 - 17x + 10$

5) $5x^2 + 17x - 12$

6) $8x^2 - 10x + 3$

Some easier ones, but factor completely:

7) $2x^2 - 50$

8) $x^2 - 5x + 6$

9) $2x^2 + 10x - 12$

10) $2x^2 - 12x - 54$

11) $16x^2 - 25y^2$

12) $3x^2 - 27$

13) $56x^4y^3 - 42x^2y^6$

14) When factored completely, $x^3 + 3x^2 - 4x - 12$ equals

(1) $(x^2 - 4)(x + 3)$

(2) $(x^2 - 4)(x - 3)$

(3) $(x - 2)(x + 2)(x + 3)$

(4) $(x - 2)(x + 2)(x - 3)$

15) The expression $x^2(x + 2) - (x + 2)$ is equivalent to:

(1) x^2 (2) $x^2 - 1$ (3) $x^3 + 2x^2 - x + 2$ (4) $(x + 1)(x - 1)(x + 2)$

Exponents and Polynomials

1) A monomial is an expression that is a number, variable or the product of a number and variables. The ***degree* of a monomial** is the sum of the exponents of the variable symbols that appear in it. Example: $3x^5y^2z^3$ is a monomial with degree 10.

2) A polynomial is an expression consisting of 2 or more monomials being added or subtracted. The ***degree* of a polynomial** is the degree of the monomial term with the highest degree. Example: $5x^3+4x^2-17x+2$ is a polynomial with degree 3.

Polynomials are closed under the operations of +, --, and •. This means that if you perform one of these operations on 2 or more polynomials, the answer is a polynomial.

3) A polynomial with 2 terms is called a binomial. A polynomial with 3 terms is called a trinomial.

4) Polynomials may be added by combining the like terms in each. *Be careful with the variables. Make sure the variables and exponents match.* x^2y^3 and x^3y^2 are not like terms.

5) To subtract polynomials, leave the first polynomial alone, change the signs of each term in the second polynomial to its opposite. Then combine like terms.

6) To multiply monomials with the same base, multiply the coefficients, keep the base, and add the exponents.

7) To divide monomials with the same base, divide the coefficients keep the base, and subtract the exponents.

8) Negative powers may be rewritten with a positive exponent by placing the base and its exponent in the opposite position of the fraction and changing it to a positive exponent. You can do the same thing if you want a negative exponent as opposed to a positive one. In later courses you may actually introduce the negative exponent as it will help you accomplish something specific.

Examples: $x^{-5}=\frac{1}{x^5}$, $\frac{2}{x^{-4}}=2x^4$, and $3x^{-7}=\frac{3}{x^7}$.

9) When dividing a polynomial by a monomial, break the expression into fractions and follow rules for dividing by monomials; when dividing polynomial by a monomial use method for long division.

10) In multiplication a "BOX" can be used to organize the binomials: $(3x-5)(2x+2)$

	3x	−5
2x	6x²	−10x
+2	6x	−10

11) You may also use the distributive property to multiply polynomials:
$3x(2x+2)-5(2x+2) = 6x^2+6x-10x-10 = 6x^2-4x-10$

Questions to try:

Perform the indicated operations and simplify.

1) When $-9x^5$ is divided by $-3x^3$, $x \neq 0$, what is the result?

2) Find the product of $\frac{1}{3}x^2y$ and $\frac{1}{6}xy^3$.

3) Simplify $\frac{(2x^3)(8x^5)}{4x^6}$.

4) Simplify $(3x^2)^4$.

5) Rewrite 2^{-3} without a negative exponent.

6) Divide and express without negative exponents: $\frac{5x^6y^2}{10x^8y}$

7) Find the result when $3a^2 - 7a + 6$ is subtracted from $5a^2 - 3a + 4$.

8) Find the product of $-3x^2y$ and $5xy^2 + xy$.

9) Find the sum of $3x^2 + 4x - 2$ and $x^2 - 5x + 3$.

10) The expression $\dfrac{9x^4 - 27x^6}{3x^3}$ is equivalent to

(1) $3x(1-3x)$ (2) $3x(1-3x^2)$ (3) $3x(1-9x^5)$ (4) $9x^3(1-x)$

11) Simplify: $2x^2 - x^2$

12) Simplify and express without negative exponents $(3c)^{-2}$.

13) Simplify $(-4a^3b)^2$.

14) Multiply and express in simplest form. $(2x-3)(x^2-4x+5)$.

Absolute Value Functions – The absolute value function is a piecewise defined function.

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

To solve absolute value equations of the form $|inside| = outside$

1) Make sure that the side of the equation with the $|\ |$, has nothing else with it.

2) Set up 2 equations and solve. The equations are:
$inside = outside$ and $inside = -outside$

Example: $|3x+4| - 2 = 6$ The absolute value is not alone on one side, so add 2 to both sides first.

$|3x+4| = 8$ Equation 1: $3x + 4 = 8$ Equation 2: $3x + 4 = -8$

$$3x = 4 \qquad\qquad 3x = -12$$

$$x = \frac{4}{3} \qquad\qquad x = -4$$

Alternate equations: Equation 1: $3x + 4 = 8$ Equation 2: $-3x - 4 = 8$

Questions to Try:

1) $|2x+3|-4=12$

2) $|2x-1|=9$

3) $|2x-5|=11$

Some more questions that relate to things in the review:

4) Tamara has two sisters. One of the sisters is 5 years older than Tamara. The other sister is 4 years younger than Tamara. The product of Tamara's sisters' ages is 70. How old is Tamara?

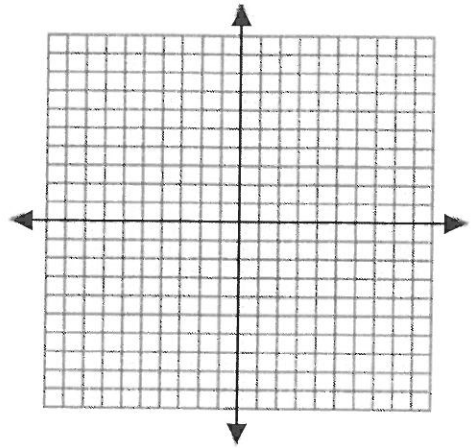
5) When the square of the third of three consecutive odd integers is added to three times the second integer, the result is 92 more than twice the first. Find the integers.

6) Solve using an appropriate method.

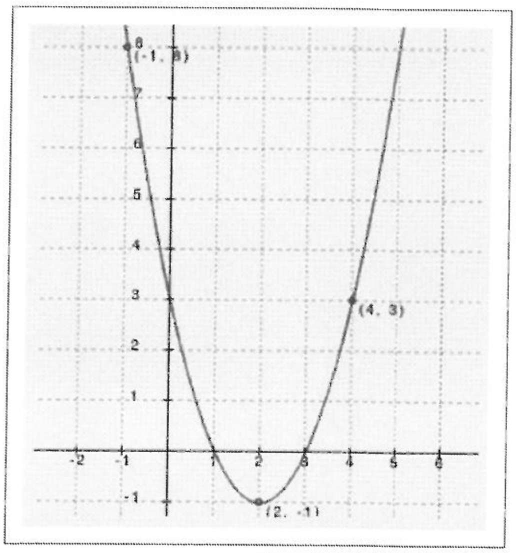
$$4(x-3)^2-8=60$$

7) Graph the function $f(x)=(x-2)(x+8)$. Be sure to find the

a) x-intercepts. b) y-intercept. c) vertex.



8) a) Fill in the missing values in the chart.



x	$f(x)$
-1	8
2	-1
4	3

b) State the x-intercepts.

c) State the y-intercept.

d) Is the leading coefficient positive or negative?

e) On what interval is the function increasing?

f) State the coordinates of the vertex.

g) Write the equation of the axis of symmetry.

h) Describe the end behavior.